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Interference Spectroscopy. Part I

KARL WILH. MEISSNER
Worcester Polytechnic Institute, Worcester Massachusetts
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THE study of the hyperfine structure of spectral lines, due to emission by the different isotopes of an element or to the interaction of the nucleus and the surrounding electrons, has given a new impetus to the application of interference spectroscopy. The resolution of the sometimes very small separations of the single components of a line requires a high resolving power, exceeding very often that of a powerful reflection grating.

Several kinds of high resolving instruments are at our disposal: *Michelson's echelon*, transmission and reflection types, *Lummer and Gehrcke's plane-parallel plate*, and, last but not least, *Fabry and Perot's interferometer*.

Each of these instruments has its inherent advantages and disadvantages. The advantage of the *echelon* is its high light intensity, simple handling, and easy reduction of the interference patterns. Furthermore, the reflection type of this apparatus is the only high resolving instrument for use in the extreme ultraviolet region for which no proper transparent medium is at our disposal. Disadvantages are its high price and the circumstance that its resolving power is fixed for a given wave-length. The *Lummer-Gehrcke plate* is comparatively inexpensive and has, if made of quartz, great advantages for ultraviolet work, but the reduction of the patterns is complicated and its resolving power is restricted by the unchangeable dimensions of the plate.

Fabry and Perot's interferometer, especially the type with fixed plane-parallel plates, is simple in principle, not too expensive, and with aluminized quartz plates is adapted for work in the whole spectral region from the infra-red to the far ultraviolet region (2000A). The resolving power of this instrument can be varied over a wide range by proper choice of the gap between the plates and the reflection power of the metal films. A resolving power corresponding to the natural width of the lines can be obtained by this instrument only. The reduction of the interference patterns is rather simple. Furthermore, it allows one to make wave-length measurements relative to one standard line alone. This circumstance makes the Fabry-Perot instrument the proper means for the determination of secondary standard wave-lengths (on the basis of the primary standard Cd 6438.4696A). The only great disadvantage of this instrument is the enormous loss of light due to the absorption of the metal reflecting films. But, however serious this may be, the advantages are so predominant that the Fabry-Perot interferometer is, at least at the present time, the preferred apparatus for hyperfine structure work.

For the region of short wave-lengths, however (Schuman and Lyman regions), the use of the Fabry-Perot instrument is impossible because of the lack of transparent substances, and the only instrument which can be used in these

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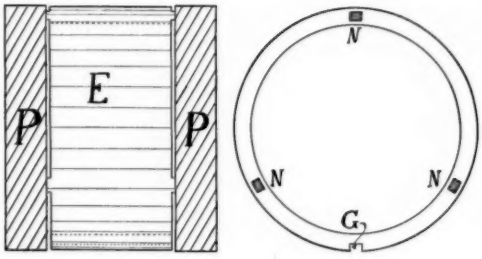

FIG. 1. Etalon spacer.

spectral regions is the reflection type of Michelson's echelon grating. In a proper arrangement, as developed by W. E. Williams, this instrument can be used in all parts of the spectrum for absolute measurements. In the future development of interference spectroscopy this latter instrument will play a very important role.

In accordance with this state of affairs we restrict our treatment to the two most important instruments, the Fabry-Perot interferometer and the reflection echelon. The first, however, will be treated more extensively because of its widespread use.

I. THE FABRY-PEROT INTERFEROMETER

§1. Construction of the instrument

The most frequently used type of Fabry and Perot interferometer is the apparatus with fixed plates, called by its inventors the etalon. The optical part of an etalon consists of two plates of quartz or glass. The plates usually have a slightly prismatic shape, the plane surfaces making an angle of several minutes (up to 30) with each other. One face of the plates is made as plane as possible, the mean deviation from the average plane preferably not exceeding 1/30 of the wave-length of the green Hg line. These faces are coated with a thin metal film of high reflecting power (silver, gold, platinum, or aluminum). The coated faces of the two plates are kept in a constant parallel position by a ring of Invar or fused quartz, shaped as in Fig. 1. The material from the faces of the etalon ring E, perpendicular to the axis of the cylinder is removed, by grinding or filing, up to the three small projections N, the untouched area of each being from 1×1 to 1.5×1.5 mm. The corresponding projections of the two faces should be situated as exactly as possible so that the line connecting their centers is parallel to the cylinder axis. Furthermore, the spacers are sometimes provided with a groove G parallel to the axis of the cylinder fitting exactly a pivot attached to the housing of the etalon. In this way a correct, reproducible position of the spacer is guaranteed. The faces of the projections are usually plane; sometimes, however, a spherical shape is preferred. The two planes determined by the faces of the projections are made as parallel as possible by grinding and polishing. In putting the plates against the etalon spacers, it is convenient to have removed the metal film from the regions of the plates which are to be in contact with the spacer. Then it is possible to check by interference patterns (using white light) whether there is optical contact between the faces of the projections and the etalon plate. This is absolutely necessary if the apparatus is to maintain its adjustment. With the aid of adjustable springs, plates, or spiral springs, attached in a proper way to the etalon housing, the plates are slightly pressed against the projections. The spring forces must act in the direction of the connecting line of corresponding projections, avoiding bending stresses. A small spring force should be sufficient to obtain a good adjustment of the plates.

The best materials for the distance rings are fused quartz or Invar (64 percent iron, 36 percent nickel), as both have very small thermal expansion coefficients.

If long exposure times cannot be avoided the etalon should be put into an airtight double-walled chamber provided with two strictly plane plates for the entrance and exit of the light. If water of constant, thermostatically controlled temperature is circulated between the walls of the chamber, the temperature can be kept constant to within 0.01°C so that the influence of varying room temperature is eliminated. This arrangement avoids also a shift of the interference patterns by the varying density of the air, which changes the refractive index and therefore the patterns. If one has to work with long etalon separations such a set-up is absolutely necessary.

§2. Theory of the interference fringes produced by the Fabry-Perot interferometer

(*a*) *Principle.*—In Fig. 2, representing a cross section through the optical part of the interferometer, *I* and *II* are the two plates of the instrument; the highly reflecting surfaces S_1 and S_2 strictly plane and parallel to one another, have a distance of separation t. Let us consider an incident plane wave, the wave normal (ray) being *R*. After refraction through the plate and partial reflection at the first face S_1 at *D*, the ray falls upon the face S_2 at *A* at an angle of incidence ϕ. At *A* the ray is split again, partly leaving the plate *II* after refraction as ray 1, partly reflected at *A* and then at *B* and refracted as ray 2. Due to multiple reflections at S_1 and S_2 in the same manner, the rays 3, 4$\cdots$ are formed, all parallel to each other, their intensity decreasing systematically with increasing number of reflections, as we shall see later. The waves going back into the first medium will not be treated in the following discussion.

The wave trains belonging to the rays 1, 2, 3$\cdots$ are split off from one original wave, are therefore coherent and can interfere. The interference patterns will be sharpest at infinity or in the focal plane of an image-forming optical system. The partial waves belonging to the partial rays 1, 2, 3$\cdots$ will at a given point of the focal plane produce a certain intensity which depends on the amplitudes of the single waves and their phase or path differences.

Let us first derive the phase difference which wave (1) and wave (2) will have at infinity. It will be the same as they have in an arbitrary common plane perpendicular to the rays. We use the plane through point *C* the trace of which (the line $CF \perp AE$), makes with *CA* the angle of refraction ψ. According to known optical principles, the phase difference of the two waves is to be found by multiplying the difference of the geometrical paths of the rays by $2\pi/\lambda$, λ being the wave-length in the particular medium, in our case λ_a for air and λ_g for glass. We get

$$\text{path difference} = AB + BC - AF$$

and therefore

$$\text{phase difference} = 2\pi[(AB+BC)/\lambda_a - AF/\lambda_g].$$

The factor 2π is usually called the order number, p, for which we find according to Fig. 2

$$p = 2t/\lambda_a \cos\varphi - 2t \tan\varphi \sin\psi/\lambda_g. \qquad (2.1)$$

Remembering that $\lambda_g = \lambda_a/n_g$ and that $n_g = \sin\varphi/\sin\psi$, where n_g is the refractive index of glass with respect to air, we transform this expression into

$$p = 2t[1/\lambda_a \cos\varphi - (\sin\varphi \sin\psi/\cos\varphi)(\sin\varphi/\lambda_a \sin\psi)], \qquad (2.2)$$

from which we finally obtain

$$p = 2t \cos\varphi/\lambda_a. \qquad (2.3)$$

This formula can be written in a second, often useful, manner using wave numbers, ν_a, defined by the relation $\nu_a = 1/\lambda_a$:

$$p = 2t \cos\varphi \nu_a. \qquad (2.4)$$

It should be stated that the refraction in the glass plate does not affect the final result at all. Without regard to the loss of light by the internal reflections in the glass plates, the whole apparatus acts just as if we had to deal with a plane parallel plate of air the faces of which are distinguished by a high reflecting power. Therefore, the Fabry-Perot interferometer is very often spoken of as the "silvered plane-parallel air plate."

In deriving the expression for the order number, p, we did not take into account the possibility of a phase change which the reflected wave may have to undergo if reflected at *A* and *B*. This phenomenon will be treated in a later part of this report (§11).

(*b*) *Geometry of the interference fringes.*—Before we go into detail regarding the intensity distribu-

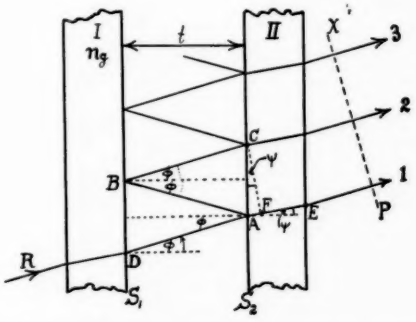

FIG. 2. Path difference between two consecutive rays.

tion we shall get approximate information about the geometric shape of the fringes situated in the focal plane of the image forming system (Fig. 3).

A phase difference equal to an integral multiple of 2π will give constructive interference of the emerging beams and a maximum of intensity; a phase difference equal to an odd multiple of π will produce destructive interference and a minimum of intensity. In other words, maximum intensity corresponds to integral values of the order number p. According to formula (2.3), p will be an integer for certain values of the angle of incidence $\varphi_1, \varphi_2, \varphi_3 \cdots$. According to Fig. 3, all rays incident under an angle φ_i and situated in planes parallel to the plane of drawing will produce in the focal plane F a bright point P_i and the same will be the case for the other rays belonging to the same angle of incidence but different azimuths, so that the result of the whole bundle of rays will be a bright circle of radius R_i. As soon as we illuminate the etalon with an extended monochromatic light source or by a convergent light beam, any angle of inclination is present; but only in those directions, $\varphi_1, \varphi_2, \varphi_3 \cdots$, for which the orders, $p_1, p_2, p_3 \cdots$, are integers, will bright interference circles be observed. The radii of these circles (or "rings") depend upon φ_i and the focal distance f of the projecting system. From Fig. 3 we can see that

$$\cos\varphi_i = f/(f^2+R_i^2)^{\frac{1}{2}} \approx (1-R_i^2/2f^2) \approx (1-D_i^2/8f^2), \quad (2.5)$$

where D is the diameter of the corresponding circle.

Replacing λ_a and ν_a by λ and ν, Eqs. (2.3) and (2.4) may be rewritten as:

$$p = (2t/\lambda)\cos\varphi = (2t/\lambda)(1-R^2/2f^2) \cdot = (2t/\lambda)(1-D^2/8f^2) \quad (2.6)$$

or in terms of wave numbers ν as:

$$p = 2t\nu\cos\varphi = 2t\nu(1-R^2/2f^2) = 2t\nu(1-D^2/8f^2). \quad (2.7)$$

These equations are the fundamental relations for the evaluation of interference patterns with respect to wave-lengths or wave-length differences or wave number differences.

First, we shall use this equation to obtain

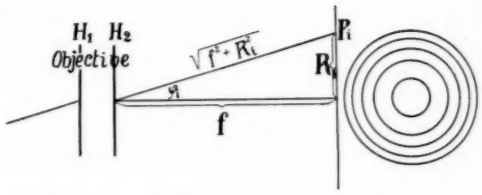

FIG. 3. Formation of circular interference fringes in the focal plane of a lens.

information about the distribution of the single interference circles and about the coordination of the order numbers and the radii. By differentiation of Eq. (2.6) we get

$$dp = -(2t/\lambda)\sin\varphi d\varphi = -(2t/\lambda)(R/f^2)dR. \quad (2.8)$$

This relation shows that the order of the fringes decreases with increasing angle of incidence φ or increasing radius R. For $dp=-1$, we get

$$\Delta R = \lambda f^2/2tR \quad (2.9)$$

which means that for larger radii, consecutive circles are closer together. Furthermore, we see that for a given arrangement (t and f constant) in the neighborhood of a certain value of R, the rings of shorter wave-length are closer together than those of longer wave-length.

According to formula (2.6), the order p is a quadratic function of R, so that to equal increments of p there correspond unequal increments of R, but equal increments of R^2. Or, in other words, the difference between the R^2 of consecutive circles is constant. Let us call the radius of the innermost ring R_0, the following R_1, and so on, R_k being the $(k+1)$th ring of the system. Then we have the relations

$$\begin{aligned} p_0 &= 2t/\lambda - R_0^2 t/\lambda f^2, \\ p_i &= p_0 - i = 2t/\lambda - R_i^2 t/\lambda f^2, \\ p_k &= p_0 - k = 2t/\lambda - R_k^2 t/\lambda f^2, \end{aligned} \quad (2.10)$$

and by subtraction:

$$(p_i - p_k) = (k-i) = (R_k^2 - R_i^2)t/\lambda f^2$$

or

$$(R_k^2 - R_i^2)/(k-i) = \lambda f^2/t = \Delta R^2,$$

or written with D values:

$$(D_k^2 - D_i^2)/(k-i) = 4\lambda f^2/t = \Delta D^2. \quad (2.11)$$

The values $R_0^2, R_1^2, R_2^2, \cdots$ (or $D_0^2, D_1^2, D_2^2 \cdots$) represent an arithmetical series of the

first order, the common difference being $\Delta R^2 = (R_k^2 - R_i^2)/(k-i)$ (or $\Delta D^2 = (D_k^2 - D_i^2)/(k-i)$). This simple relation will prove very useful for measuring wave-lengths and wave-length or wave number differences.

(*c*) *Order number of the center of the ring system.*—Although, primarily, we attributed integral orders only to the bright fringes, we may assign to each point of the interference field on the focal plane an order number which will be a fractional number between the maxima. For certain calculations, it is very convenient to attribute to the center of the ring system ($\varphi = 0$, or $R = D = 0$) a (generally) non-integral order which we may call P. According to our fundamental formulae (2.3) and (2.4) the following relation holds:

$$P = 2t/\lambda \quad \text{or} \quad P = 2t\nu. \tag{2.12}$$

The order of the center P can be expressed by the integral order of the first bright fringe p_0 and a positive fractional part, ϵ, or by the order of the $(k+1)$th fringe as $P = p_k + k + \epsilon$. The knowledge of the fractional part ϵ sometimes facilitates the calculations, as we shall see later, and it can easily be calculated if the radii or diameters of two rings are known: According to Eqs. (2.12) and (2.6) we have

$$P = 2t/\lambda,$$
$$p_k = 2t/\lambda - (t/\lambda f^2) R_k^2.$$

Therefore

$$\epsilon = P - p_k - k = (t/\lambda f^2) R_k^2 - k, \tag{2.13}$$

and using formula (2.11) we finally get

$$\epsilon = \frac{R_k^2}{(R_k^2 - R_i^2)/(k-i)} - k, \qquad = \frac{R_i^2}{(R_k^2 - R_i^2)/(k-i)} - i. \tag{2.14}$$

It is clear that instead of using the R values we can calculate ϵ from the measured D values directly, the formula being

$$\epsilon = \frac{D_i^2}{(D_k^2 - D_i^2)/(k-i)} - i. \tag{2.15}$$

Putting the denominator equal to ΔR^2 (or ΔD^2) we may write, more briefly: Fractional part

$$\epsilon = R_i^2/\Delta R^2 - i = D_i^2/\Delta D^2 - i. \tag{2.16}$$

For the innermost circle ($i = 0$) we have:

$$\epsilon = R_0^2/\Delta R^2 = D_0^2/\Delta D^2. \tag{2.17}$$

§3. Proper calculation of the fractional part ϵ by the method of least squares

In most papers about interferometric work little or nothing is said about the method used for obtaining the value of the fractional part $\epsilon = D_0^2/\Delta D^2$. If only two circles are at our disposal, their diameters being D_0 and D_1, there is only one possible method of calculation: The combination of the squared diameters yields $\Delta D^2 = D_1^2 - D_0^2$ and D_0^2 is taken from the directly measured D_0.

But usually and preferably, in high precision work and in the case of greater etalon separations t, numerous diameters (say n) D_0, D_1, D_2, $\cdots$ D_{n-1} are measured. As the squares of these values form an arithmetical series of the first order, $(D_{i+1}^2 - D_i^2)$ being the common difference ΔD^2, it is necessary to calculate ΔD^2 in a proper way. The arithmetical mean of the single ΔD^2 will not give the correct value because only D_0^2 and D_{n-1}^2 have to be known to get it, so that the middle members of the series would have no influence. On the other hand, the measured D_0^2 is not the only value that can be derived from the set of observations. With the correct value of ΔD^2 known, we can calculate different values for D_0^2, for instance $D_0'^2 = D_1^2 - \Delta D^2$, $D_0''^2 = D_2^2 - 2\Delta D^2 \cdots$, etc., which have to be considered, but with different weights, by calculating the mean of D_0^2. If we allow a slight approximation, a very simple scheme can be set up by the method of least squares with the aid of which it is easy to obtain the average values for D_0^2 and ΔD^2 and from them the fractional part ϵ. The error lies in the assumption that the set of the D_k^2 values and not the D_k values are the observed quantities.

Between the squares of the diameters and the common constant difference exists, according to (2.11), the relation:

$$D_k^2 = D_0^2 + k\Delta D^2$$

or, putting $D_k^2 = Y$, $D_0^2 = A$, $k = X$, and $\Delta D^2 = B$ we can write the relation in the form $Y = A + BX$.

The constants A and B have to be determined so that the sum of the squares of the deviations

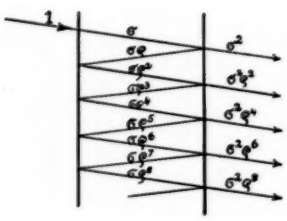

FIG. 4. Amplitudes of the transmitted waves due to multiple reflection.

$(Y_{calc}-Y_{obs})$ become a minimum. In our case the equations become simple because the observations of X are taken in equal intervals, each being unity.

A very convenient treatment of such cases is given by H. M. Roeser.[1] Referring to this paper, we restrict ourselves to the reproduction of the results and to the illustration of the method by an example.

Let X_m be the average of X (our indices k), Y_m the average of $Y(D_k^2)$, then the constants A and B are given by

$$B=6\frac{(n-1)(y_n-y_1)+(n-3)(y_{n-1}-y_2)+\cdots}{n(n^2-1)}$$

$$=\frac{\Sigma}{n(n^2-1)/6}$$

and

$$A=Y_m-BX_m$$

n being the number of each observation, in our case equal to $(k-1)$.

The following example for a set of six rings, taken at random from an earlier work, may show the manner of calculation, using a clearly arranged scheme as given by Roeser:

No. of Observ.	Number of Ring X	Square of Diameter Y	I Y_n-Y_1 $Y_{n-1}-Y_2$ ⋯	II $n-1$ $n-3$ ⋮	I×II	$B=\frac{\Sigma}{(n/6)(n^2-1)}=26.47$ $A=Y_m-BX_m=25.08.$
1	0	24.86				
6	5	157.28	132.42	5	662.10	
2	1	51.75				
5	4	130.96	79.21	3	237.63	
3	2	78.00				Therefore: $D_0^2=25.08$ $\Delta D^2=26.47$
4	3	104.69	26.69	1	26.69	
	$X_m=2.5$	$Y_m=91.26$			$\Sigma=926.42$	

$$\epsilon=\frac{D_0^2}{\Delta D^2}=0.947.$$

[1] H. M. Roeser, Sci. Pap. Bur. Stand. **16**, 363 (1920).

A shorter method is the method of approximation, using proper approximate values for A and B and dealing with the differences $y=Y_{obs}-Y_{calc}$, and the corrections a and b for the approximate values of A and B, which are connected by the relation $y=a+bx$. It will be sufficient to illustrate the method for the same example. With the approximate values $B=26.50$ and $A=25.00$ we get the following scheme:

X	Y_{obs}	Y_{appr}	Y	I Y_n-Y_1 ⋮	II $n-1$ $n-2$ ⋮	I×II	
0	24.86	25.00	−.14				$a=-\frac{1.08}{35}=-.030$
5	157.28	157.50	−.22	−.08	5	−.40	
1	51.75	51.50	+.25				$b=0.007+0.077=+0.084$
4	130.96	131.00	−.04	−.29	3	−.87	
2	78.00	78.00	±.00				
3	104.69	104.50	+.19	+.19	1	+.19	

$X_m=2.5$ $\quad Y_m=+0.007$ $\quad \Sigma=-1.08$

$A=\Delta D^2=26.50-0.03=26.47$

$B=D_0^2=25.00+0.08=25.08.$

This method of calculation should be applied in each case when we have to deal with high precision work. The method is to be preferred to graphical methods because of its higher accuracy.

§4. Intensity distribution in the interference patterns

From the geometric discussion, we can get only a rough idea about the shape of the interference patterns, but we do not see the influence of the multiple reflected rays. However, by comparing the action of the plane-parallel plate with that of a grating we can easily see what will happen. A grating splits an incident plane wave into several partial waves which will interfere at infinity. Only at spots where the waves, coming through two consecutive slits, have a path difference of $p\lambda$, p being an integer, will there be constructive interference. The use of a great number of slits has two effects: first it increases the intensity and second it produces sharper lines. We have to expect the same effect with the interferometer, but it must be taken into account that here the intensities of two consecutive beams are not equal but decrease systematically with the number of reflections. For a clear understanding of the action of an interferometer, the knowledge of the intensity

distribution is very important. We therefore give a short derivation of the expression for it.

Let ρ and σ be the reflection and transmission coefficients of the surfaces S_1 and S_2, the physical properties of which are supposed to be identical. An incident wave with the amplitude A therefore will split into two waves with the amplitudes ρA and σA. In Fig. 4 the incident wave has the amplitude 1 and it is easy to be seen that the amplitude of the kth wave train will be $\sigma^2 \cdot \rho^{2(k-1)}$. Replacing σ^2 and ρ^2 by the experimentally more important intensity coefficients s and r, called transmission and reflection power, we write the amplitude of the kth train as $s \cdot r^{(k-1)}$. The light vibrations excited by the different partial waves at a certain point of the focal plane of the projecting lens, therefore, will be:

$$s \cdot e^{i\omega\tau}, \quad sre^{i(\omega\tau - 2\pi p)}, \quad sr^2 e^{i(\omega t - 4\pi p)} \cdots,$$

where $\omega/2\pi$ is the frequency of the light and τ the time; p is the order number, $2\pi p$ the phase difference between consecutive beams (see §2). The sum of the partial vibrations will yield the resulting vibration. In the case of the interferometer, we have to deal with a practically infinite number of partial trains because of the small angles of incidence and the relatively large area of the plates. In this case the summation is very simply performed: The resultant vibration may have the amplitude A, then we get

$$\begin{aligned} Ae^{i\omega\tau} &= se^{i\omega\tau} \sum_1^\infty {}_k\, r^{(k-1)} e^{-i2\pi p(k-1)} \\ &= se^{i\omega\tau}(1 + re^{-2\pi pi} + r^2 e^{-2(2\pi p)i} + \cdots) \\ &= se^{i\omega\tau}/(1 - re^{-2\pi pi}). \end{aligned} \tag{4.1}$$

The observable intensity, I, is equal to $A \cdot \bar{A}$, where $\bar{A}$ is the complex conjugate of A. Therefore

$$\begin{aligned} I &= s^2/(1 - re^{-2\pi pi})(1 + re^{+2\pi pi}) \\ &= s^2/(1 + r^2 - 2r\cos 2\pi p). \end{aligned} \tag{4.2}$$

TABLE I.

$I : I_{max}$ / r	0.50	0.20	0.10	0.04	I_{min}/I_{max}
0.9	0.0169	0.0336	0.0505	0.0831	0.00277
0.8	0.0357	0.0718	0.1089	0.1845	0.01235
0.7	0.0574	0.1167	0.1808	0.3413	0.03113
0.6	0.0831	0.1726	0.2820	—	0.0625

Replacing $\cos 2\pi p$ by $(1 - 2\sin^2 \pi p)$, this expression can be written:

$$I = s^2/((1-r)^2 + 4r\sin^2(\pi p)). \tag{4.3}$$

$$I = \frac{s^2/(1-r)^2}{1 + (4r/(1-r)^2)\sin^2(\pi p)}$$

For p = integer, we get

$$I_{max} = s^2/(1-r)^2.$$

For p = integer $+\frac{1}{2}$, we get

$$I_{min} = s^2/((1-r)^2 + 4r) = s^2/(1+r)^2.$$

As p is connected with the angle of incidence or the radius R of the interference rings according

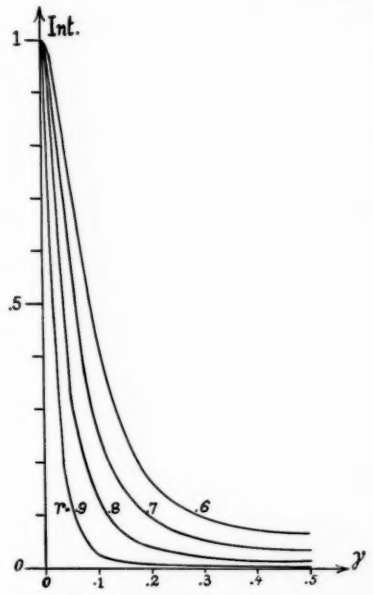

FIG. 5. Intensity distribution depending upon reflection power.

to formulae (2.3) and (2.6), $p = 2t\cos\varphi/\lambda = 2t(1 - R^2/2f^2)/\lambda$, we see that along a diameter of the ring system, the intensity fluctuates between these two extreme values I_{max} and I_{min}.

A discussion of the intensity expression shows that the bright zones of a ring are very sharp compared with the dark ones and that the rings become sharper with increasing reflecting power r. To show this behavior, there is calculated in Table I what change $\pm\gamma$ the order p has to undergo to depress the intensity to $\frac{1}{2}$, $\frac{1}{5}$, $\frac{1}{10}$ and $\frac{1}{25}$

of the maximum intensity. The last row gives the relative minimum intensity which we get for points in the middle between the orders ($\gamma=\pm 0.5$). Fig. 5 represents these calculations and shows strikingly the influence of the reflection power on fringe sharpness.

The maximum intensity $I_{max}=s^2/(1-r)^2$ depends on both the transmission and absorption power. In the case of an ideal, non-absorbing medium, $(r+s)=1$, therefore $s^2=(1-r)^2$. The intensity of a fringe then would be the same as we would get without the interferometer. But if we have to deal with metals, high reflection power is always connected with absorption and we have the relation $r+s=1-a$, where a is the absorption power. Thus the maximum intensity of the fringes becomes smaller than that of the incident light.

Summarizing, we can say that the sharpness of the interference fringes depends upon the factor $4r/(1-r)^2$ and that the loss of light by absorption is increased by the factor $s^2/(1-r)^2$. For getting sharp lines, the first factor has to be increased up to a certain value by increasing the thickness of the metal film. For silver films, usually, we go up to a reflection power of 90 percent of that which is obtained with a film of about 40 mμ thickness. The transmission then is only about 3 percent and the absorption 7 percent. Therefore, the maximum intensity we can expect is only about 9 percent of the intensity we would get without an interferometer. Therefore, it is to be understood that the use of an interferometer requires long exposure times, in our case about ten times longer than without an interferometer.

For problems concerning sharpness of interference fringes or the width of spectral lines, an approximation of formula (4.3), holding in the neighborhood of the maximum intensity, is sometimes useful.

Let p in (4.3) be an integer and γ a small fractional part of the adjacent order, then the intensity for the order $(p\pm\gamma)$ will be given by

$$I_\gamma=I_{max}(1-r)^2/((1-r)^2+4r\sin^2(\pi\gamma)). \quad (4.4)$$
$$=I_{max}/(1+\eta^2\sin^2(\pi\gamma)),$$

where

$$\eta^2=4r/(1-r)^2. \quad (4.5)$$

For small values of γ we may use the first approximation

$$I_\gamma=I_{max}/(1+\eta^2\pi^2\gamma^2). \quad (4.6)$$

A convenient measure for the sharpness of the fringes is the so-called *apparatus half-width*, that is the total width $2\gamma_h$ of the intensity curve for an intensity equal to $I_{max}/2$. For this case, the approximation yields

$$I_{max}/2=I_{max}/(1+\eta^2\pi^2\gamma^2)$$

or

$$\gamma_h=1/\pi\eta=(1-r)/2\pi\sqrt{r}. \quad (4.7)$$

Therefore, the apparatus half-width, measured in order numbers is

$$2\gamma_h=(1-r)/\pi\sqrt{r}. \quad (4.8)$$

The next approximation is

$$2\gamma_h=\frac{(1-r)}{\pi\sqrt{r}}+\frac{1}{24}\left(\frac{(1-r)}{\sqrt{r}}\right)^3.$$

Other expressions for the half-width will be given in §5.

§5. The interferometer as a spectroscope

In all our considerations, we have supposed that only strictly monochromatic light waves are incident on the interferometer. But strictly monochromatic light is a physical impossibility. Each spectral line has a certain intensity distribution due to damping by radiation (finite lifetime of the atom levels), due to the temperature of the light source (Doppler effect) or due to effects of electric and magnetic fields, including fields of the neighboring atoms. Although we know this, let us continue in our considerations supposing infinitely narrow lines to be a reality.

Our next question is: How would the interference patterns change if the wave-length of the monochromatic light were to change by a small amount $\Delta\lambda$ from λ to $\lambda\pm\Delta\lambda$, or the wave number ν from ν to $\nu\mp\Delta\nu$? We will find the answer from our fundamental relations (2.3), (2.6) and (2.7). Considering the order p as constant and λ, ν, φ and R as variable we get by differentiation:

$$\pm p\Delta\lambda=\mp 2t\sin\varphi\Delta\varphi=\mp 2tRdR/f^2$$

and $$\mp\Delta\nu=\mp(pR\Delta R)/2tf^2. \quad (5.1)$$

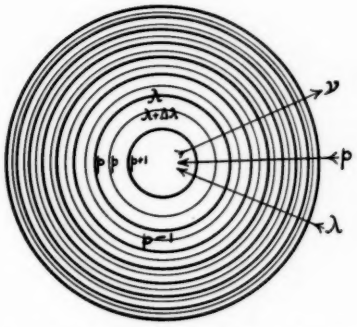

FIG. 6. Interference system of two close lines.

These equations show: Increasing λ or decreasing ν produces a shrinking of the ring system, decreasing λ or increasing ν causes a swelling of the rings.

Supposing the original line λ and the "shifted line" $(\lambda+\Delta\lambda)$ may be incident at the same time, then we would see a double ring system and if we could change $\Delta\lambda$ continuously we would observe the rings belonging to $(\lambda+\Delta\lambda)$ to be wandering between the fixed rings of λ. Let us call the fixed line λ_1, and the changeable one λ_2: At the beginning of the performance, say $\lambda_1=\lambda_2$, so that only one ring system could be observed. Then with the slow continuous change of λ_2 the λ_2 rings disengage from the λ_1 rings, the radii decreasing more and more until the two ring systems coincide again. In this manner, a certain λ_2 ring with the order p first would disengage a λ_1 ring of the order p, and finally coincide with a λ_1 ring of the order $(p+1)$ (Fig. 6).

The change in wave-length necessary to shift the ring system by the distance of consecutive orders is called "*spectral range*" $\langle\Delta\lambda\rangle$. As soon as λ_2 is changed by more than $\langle\Delta\lambda\rangle$ the λ_2 ring of the order p penetrates the λ_1 ring of the order $(p+1)$ and the rings move towards the λ_1 order $(p+2)$. The orders "overlap" each other.

From the fundamental formula (2.3) or (2.6), we can derive a proper expression for the spectral range $\langle\Delta\lambda\rangle$. If the λ_2 ring of order p is in coincidence with the λ_1 ring of order $(p+1)$ we have, φ or R being the same,

$$(p+1)\lambda_1=p\lambda_2$$

or

$$\langle\Delta\lambda\rangle=(\lambda_2-\lambda_1)=\lambda_1/p=\lambda_1^2/2t.$$

Replacing λ_1 by λ, we state:

$$\textit{Spectral range:}\quad \langle\Delta\lambda\rangle=\lambda/p=\lambda^2/2t. \tag{5.2}$$

If we express it in wave numbers we get:

$$\langle\Delta\nu\rangle=|\langle\Delta\lambda\rangle|\lambda^2|=1/2t. \tag{5.3}$$

As we see, the Fabry-Perot apparatus acts as a spectral apparatus, separating the light of different wave-lengths. In the case of a spectroscope, we use the angular dispersion D_a, defined by the expression $\Delta\alpha/\Delta\lambda$ where $\Delta\alpha$ is the change of the angle of deviation α for a corresponding change of wave-length $\Delta\lambda$. In the case of a spectrogram, on which the separation of lines is determined by linear displacements, s, we use the linear displacement, D_l, defined as $\Delta s/\Delta\lambda$.

It is clear that also in the case of the interferometer we can use both expressions. By differentiating the proper form of our fundamental formulae (2.3) or (2.6), we easily derive the following expressions: angular dispersion

$$|D_a|=|\Delta\varphi/\Delta\lambda|=p/(2t\cdot\sin\varphi)=1/(\lambda\cdot\tan\varphi) \tag{5.4}$$

and linear dispersion

$$|D_l|=|\Delta R/\Delta\lambda|=pf^2/2tR=f^2/R\lambda. \tag{5.5}$$

Both D_a and D_l are independent of the value of the etalon gap. Whether t is 3 mm or 200 mm, the shift $\Delta\varphi$ or ΔR for a given $\Delta\lambda$ will be the same in the neighborhood of a given φ or R. The dispersion increases with decreasing φ and R, reaching infinity for $\varphi=R=0$. Whether two close lines will be better resolved by means of a larger or smaller gap will be treated in the next paragraph.

The expression for the spectral range can be used for another formulation of the apparatus half-width of the interference fringes as developed in §4 (4.8). Because a change of the order p by $\pm\gamma$ corresponds to a change in wave-length $\Delta\lambda=\mp(\lambda^2/2t)\cdot\gamma=\mp\langle\Delta\lambda\rangle\cdot\gamma$, or to a change in wave numbers $\Delta\nu=\pm(1/2t)\cdot\gamma=\pm\langle\Delta\nu\rangle\cdot\gamma$, formula (4.8) can be written:

apparatus width;

$$A_\nu=2\gamma_h/2t=2\gamma_h\cdot\langle\Delta\nu\rangle\ \text{cm}^{-1}$$

$$=\frac{1}{2\pi t}\frac{(1-r)}{\sqrt{r}}=\frac{\langle\Delta\nu\rangle}{\pi}\frac{(1-r)}{\sqrt{r}}\ \text{cm}^{-1} \tag{5.6}$$

and corresponding

$$A_\lambda = \frac{\lambda^2}{2\pi t}\frac{(1-r)}{\sqrt{r}} = \frac{\langle\Delta\lambda\rangle}{\pi}\frac{(1-r)}{\sqrt{r}}\ \text{A}. \qquad (5.7)$$

These formulae are important if the interferometer fringes are used for the determination of the half-width of not infinitely narrow spectral lines. It is obvious that the observed total half-width is the sum of the real half-width of the spectral line and the half-width due to the interferometer. Therefore, for a determination of the real half-width the latter has to be known.

§6. Resolving power of the Fabry-Perot interferometer

Resolving power of a spectroscope is defined by the expression $R=\lambda/\Delta\lambda$, where $\Delta\lambda$ is the wave-length difference of two infinitely sharp spectral lines of equal intensity which can be seen just separated by the apparatus. However sharp the spectral lines may be, the apparatus, by diffraction and interference effects, produces an intensity distribution, characteristic for the apparatus. Thus, two close lines will give an intensity distribution which is the superposition of the two separate intensity curves.

In the case of a grating with N rulings, between two main maxima (consecutive orders of the spectra) there are situated $(N-1)$ minima of intensity zero and $(N-2)$ intermediate maxima of very small intensity. In accordance with the Rayleigh criterion, we assume that two lines can be separated if the maximum of one line just coincides with the first minimum of the other. The resolving power of a grating is then $R=pN$, where p is the order of the spectrum and N the total number of rulings.

Regarding the interferometer, we assumed that an infinite number of wave trains will interfere. This is only correct for rays incident perpendicular to the plates, but, since the angle of incidence is usually very small our intensity formula will be a good approximation. As we saw in §4, there is no fluctuation of the intensity between consecutive orders, as in grating spectra and therefore the Rayleigh criterion cannot be applied immediately. We can, however, translate it so that it fits also for our case. As it can be derived by diffraction theory, the intensity distribution of the grating spectrum of two identical lines in the Rayleigh position is so that the intensity midway between the peaks of the intensity curve is 0.8106 of the maximum intensity. Therefore, formulating Rayleigh's condition corresponding to this fact, we will assume that two lines of equal intensity are regarded as resolvable, if their maximal intensity and the minimum intensity between them have the ratio 1 : 0.8. This approximate value is close enough for our purpose.

From §4, we know the intensity distribution of an infinitely fine line as a function of the order p. Therefore it is convenient to express the resolving power R also in terms of p. For small angles of incidence, our fundamental formula becomes $p\lambda=2t$ and, by differentiation, we get $p\Delta\lambda+\lambda\Delta p=0$. Thus we get

$$\text{Resolving power}\quad R=|\lambda/\Delta\lambda=|p/\Delta p|.$$

The interpretation of this relation is the following: measuring the shift of the one line (λ_2 in §5) in terms of the order number of the other line, we have to have a shift of at least $\langle\Delta p\rangle=p/R$ to obtain the limit of resolution. Figure 7 shows the superposition of the two intensity curves, corresponding to a reflection power of 60 percent. According to our statement, the limit of resolution is determined by the relation

$$I_{\max} : I_{\min} = 1 : 0.8 = 1.25.$$

Using the designation of Fig. 7, we can see that

$$I_{\max} = I(p_0) + I(p_0 \pm \langle\Delta p\rangle)$$

and

$$I_{\min} = 2I(p_0 + \langle\Delta p\rangle/2)$$

where p_0 is a whole number. With the expression (4.3) for the intensity, in which we replace $4r/(1-r)^2$ by η, the condition for resolution becomes

$$(1+\eta\sin^2(\pi p_0+\langle\Delta p\rangle/2)) = 2.50(1+\eta\sin^2(\pi p_0+\langle\Delta p\rangle))$$

or, since p_0 is an integer

$$(1+\eta\sin^2(\pi\langle\Delta p\rangle/2)) = 2.5(1+\eta\sin^2(\pi\langle\Delta p\rangle).$$

Introducing the resolving power R for $\langle\Delta p\rangle$ (relation $R=p/\langle\Delta p\rangle$) we find:

$$1+\eta\sin^2(\pi p/2R) = 2.5(1+\eta\sin^2(\pi p/R). \qquad (6.1)$$

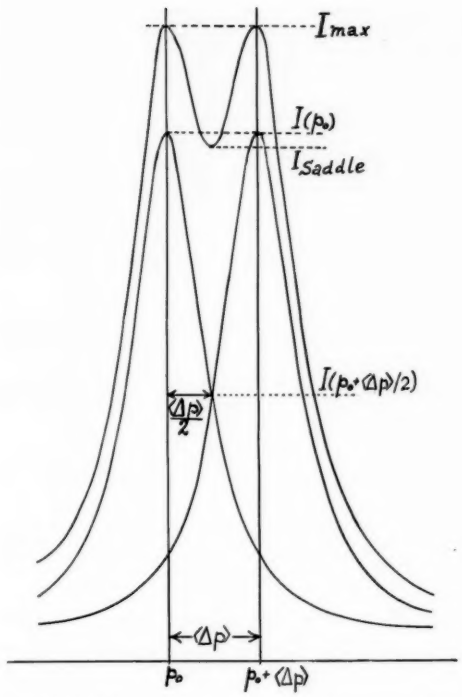

FIG. 7. Superposition of the intensity curves of two just resolved spectral lines.

A first approximation to this equation is obtained by putting $\sin x = x$

$$\pi p/2R = ((1+1.5^{\frac{1}{2}})/2)^{\frac{1}{2}}/\eta^{\frac{1}{2}}. \qquad (6.2)$$

Replacing η by its value, we find for the resolving power R

$$R = 2.98 pr^{\frac{1}{2}}/(1-r). \qquad (6.3)$$

In comparison with the formula for the grating (R = number of lines times order), our formula can be written in the form

$$R = \text{effective number of rays} \times \text{order},$$

where the effective number is a simple function of the reflection power:

$$N_{\text{eff}} = 2.98 r^{\frac{1}{2}}/(1-r).$$

Neglecting the fact that by overlapping of the two intensity curves the maxima of the resultant intensity curve are slightly increased (at $p = p_0$ and $p = p_0 + \langle\Delta p\rangle$), *Childs* derives an expression which yields as effective number of rays the value $2.56 r^{\frac{1}{2}}/(1-r)$, about 15 percent smaller than the more exact value above.

The definition of resolving power is arbitrary to a certain degree. If for the analysis of an intensity curve a photometer is applied, the ratio $I_{\min} : I_{\max}$ may be 0.9 instead of 0.8 so that a higher resolving power would result ($R = 3.5 pr^{\frac{1}{2}}/(1-r)$).

Therefore, the first approximation given above, is sufficient for all practical cases.

§7. Adjustment of the interferometer

If the spacers are correctly polished and if the reflecting surfaces of the plates are in fairly good contact with the polished surfaces of the spacer, the final adjustment of the instrument is very easily performed. First the contact between spacer and plates is tested by means of Newton fringes in white light; no colors should be present or at least only the dark violet of first order if a slight pressure is applied. (Many constructions of interferometer housings do not allow this very important test and therefore give rise to difficulties.) Then the observer, looking with the naked eye through the instrument at an extended homogeneous light source, will see the interference fringes if his eye is focused for infinity and if he looks in a direction perpendicular to the surface of the plates. (This direction is given by observing the image of the observing eye.) Shifting his eye from left to right he then may see that the diameters of the rings are continuously changing, say increasing. This indicates that the distance between the plates at the right side is too great. The spring force at the right side has then to be increased slightly until no change of the ring diameters can be seen. The same test is then made by moving the eye along a vertical diameter, adjusting in this case the tension of the upper spring. After the final adjustment, all springs should exert about the same force.

For high precision work, the adjustment with the naked eye must be followed by a fine adjustment with a telescope of small aperture but sufficient magnifying power. An aperture of about 8 mm and a magnifying power of 10 diameters is suitable. The best way is to use a special adjustment accessory consisting of a light source in front of a collimator, a telescope,

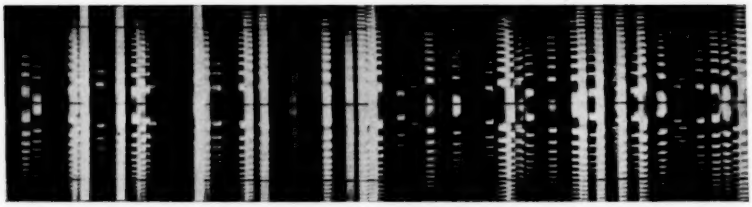

FIG. 8(a) Argon spectrum, 10 mm etalon (positive).

and a slide arrangement which allows one to shift the whole arrangement in both vertical and horizontal directions. The space between collimator and telescope must be so ample that the interferometer can be placed between them without difficulty. Two parallel cross-hairs, the interval of which can be adjusted, allow now an exact test of the constancy of the ring diameters if the collimator arrangement is moved in a horizontal or a vertical direction, as one did it originally with the naked eye.

It is clear that, in a similar way, the telescope-collimator arrangement can be fixed and the interferometer be moved. This way is advisable if, for adjusting interferometers with very long distances, liquid-air-cooled light sources (krypton tubes) have to be employed.

A very sensitive test without cross-hairs is the observation of a ring system in which a ring just begins to emerge from the center. The change of the intensity distribution of this spot can be judged with a very high accuracy without cross-hairs. If the interferometer is placed in an air tight chamber, provided with windows, the pressure in the container can easily be chosen so that the ring system shows the sensitive shape.

For adjustments with smaller distances, a neon tube is very convenient; for long distances (my experience goes up to 200 mm) a krypton tube, cooled by liquid air, is necessary.

§8. Crossing of interferometer and spectral apparatus

The investigation of light composed of a great number of wave-lengths requires resolution of the complex ring system obtained by the superposition of the patterns of the single lines. This is usually performed by crossing the high resolving instrument (etalon, echelon, etc.) with a spectral apparatus of smaller resolving power,

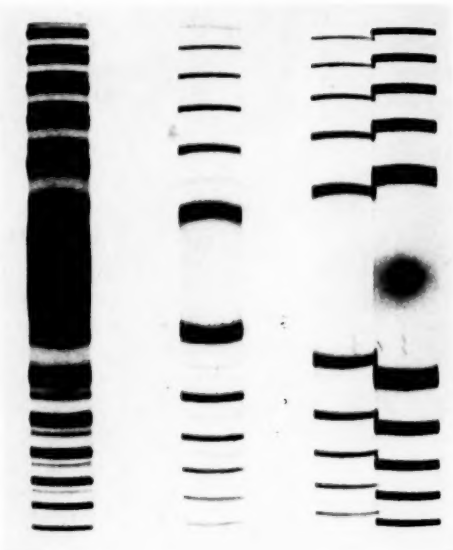

FIG. 8(b) Helium spectrum, 3 mm etalon (negative).

but sufficient dispersion. Either a grating or a prism apparatus can be used, but both have to be of the stigmatic type. If there is a slight astigmatism, the apparatus has to be adjusted in such a way that lines parallel to the direction of dispersion appear sharp in the focal plane.

Two different methods are in use:

(a) The interferometer is put at a certain distance in front of the spectrograph slit and the interference fringes are projected on the slit by an achromatic lens or a mirror system. The inclination of the interferometer plates usually is so adjusted that the middle line of the rather wide slit cuts the concentric ring system along a diameter. The interferometer should be put at such a distance that an image of the etalon aperture is formed in the exit pupil of the

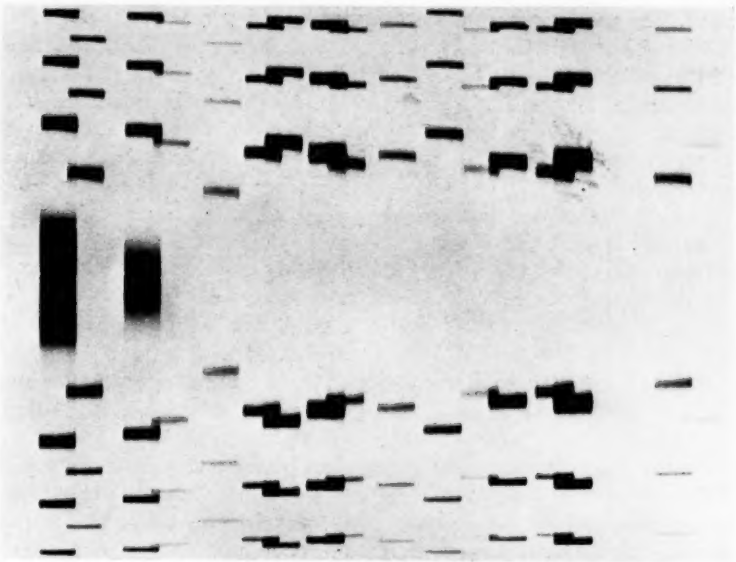

FIG. 8(c) Neon spectrum, 3 mm etalon (negative).

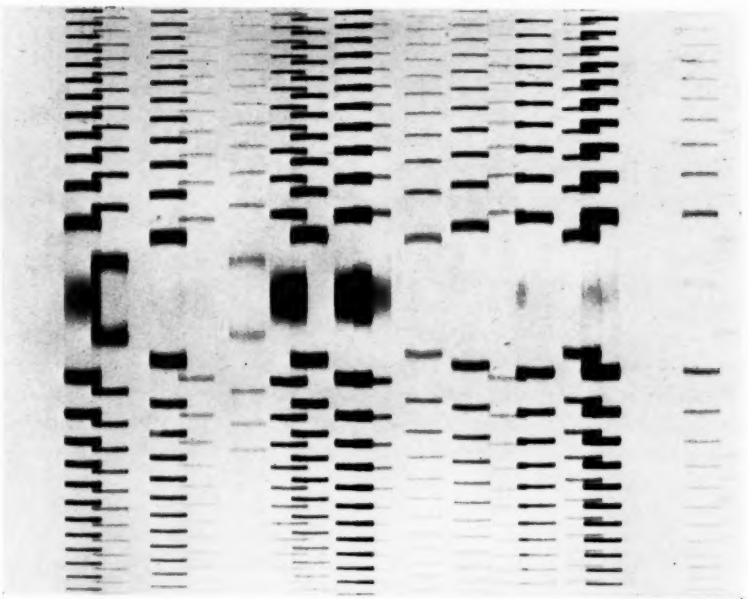

FIG. 8(d) Neon spectrum, 10 mm etalon (negative).

spectral apparatus. The part of the light source which has to be investigated is focused on the interferometer by means of another lens. This precaution guarantees a symmetric intensity distribution in the interference fringes.

With such a set-up, we obtain photographs such as those reproduced in Fig. 8. The different spectral lines, separated by the action of the prism or grating, are crossed by the bright interference patterns which the slit of the spectroscope has cut out of the whole pattern projected on the slit. If the adjustment is made so that the center of the ring system coincides with the center of the slit, we are able to measure the diameters of the interference rings of the different lines, and determine the important data we need for deducing wave-lengths or wave-length differences from them.

For one reduction method (§10), it is necessary to know the true size of the diameters as they appear on the spectroscope slit. In this case, the diameters taken from the plate have to be divided by the magnification factor of the spectral apparatus. This is defined by $k=S:I$, where S and I are the respective separations of two lines (perpendicular to the slit) on the slit and on the photograph. For the determination of k, the slit cheeks bear a metal plate, provided with a broader slit parallel to the apparatus slit and with three thin metal wires or silica fibers perpendicular to the slit. The middle wire is adjusted in the axis of the collimator, the two others at a distance of 5 to 10 mm. Thus, the spectral lines are traversed by three dark lines, the images of the wires (see Fig. 8(a)). From the measurement of their separation and that of the wires themselves, we find the factor k for each line. It is convenient to compile the different values, taken from one plate, in a graph for k as a function of λ and to draw a smooth curve through the points from which the final mean values for k may be taken.

(b) In the second method, the interferometer is placed between collimator lens and dispersing system in the parallel light beam emerging from the collimator. This method has certain advantages and disadvantages if compared with method (a). It gives a higher intensity and a very much easier and more compact set-up. On the other hand, it is to be seen that in method (b) the different zones of the slit are illuminated by different parts of the light source so that we have to be sure that all these parts emit the same wave-lengths. In method (a), usually only a smaller area of the interferometer will be used so that even with not first-class plates a region of perfect parallelism can be found and used. Method (b) affords plates of at least the diameter of the spectrograph optics and, therefore, first-class plates and perfect adjustment are needed. It is clear that, at the cost of intensity losses, the definition of the fringes can be improved by suitable diaphragms if first-class plates are not at our disposal. Another disadvantage of method (b) is the appearance of secondary images, produced by reflection at the uncoated surfaces of the plates. Although these usually are slightly prismatic, their angles being 10 to 30 minutes, and their refracting edges make an angle of 45° with the direction of dispersion, it is sometimes unavoidable that the secondary images of strong lines disturb the patterns of the weaker neighbor lines. If the interferometer is kept in an airtight chamber, covered with plane plates, the number of secondary reflection patterns will be increased, but, in general, one will put up with them because of the other advantages. The main advantage of method (b) is that it allows one to make use of the expensive, generally excellently corrected camera lens system for the projection of the interference fringes. The long focal length of these lenses very often brings the efficiency of an interferometer set up to its optimum.

A third method is sometimes used: In the production of the collimator axis, in front of the slit, a separate collimator arrangement is set up in such a way that parallel light passes the interferometer, and a projecting lens forms the images of the fringes at the plane of the slit of the spectrograph. Thus the slit of the auxiliary collimator is projected on the spectrograph slit, the secondary images, however, displaced by the prismatic shape of the plates, cannot enter

TABLE II. *Magnesium line 5528A, etalon gap t=36.008 mm.*

		ΔH		ΔH		ΔH		ΔH		ΔH	
Mg^{24}	*34.6*	43.9	*78.5*	43.9	*122.4*	43.8	*166.2*	43.8	*210.0*	43.5	*253.5*
ΔV_1	11.2		11.7		10.8		11.2		11.1		11.0
Mg^{25}	*45.8*	44.4	*90.2*	43.0	*133.2*	44.2	*177.4*	43.7	*221.1*	43.4	*264.5*
ΔV_2	11.5		10.2		11.3		11.6		11.0		10.8
Mg^{26}	*57.3*	43.1	*100.4*	44.1	*144.5*	44.5	*189.0*	43.1	*232.1*	43.2	*275.3*

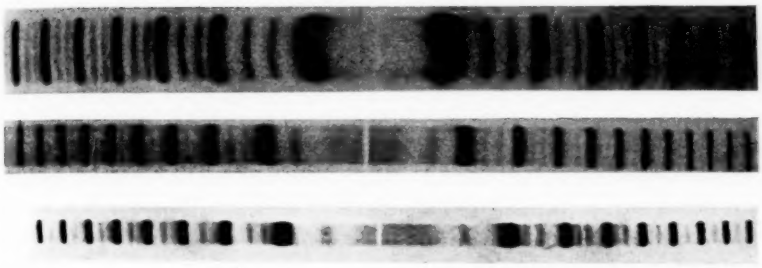

FIG. 9. (a) Magnesium λ8806A, 3.6 cm etalon. (b) λ5528A, 4.2 cm etalon. (c) λ5528A, 3.6 cm etalon.

the spectrograph. Thus, the disturbance of the secondary images is avoided.

§9. Measurement of small wave-length differences or intervals

The determination of wave-length differences between close lines is the simplest interferometric measurement. All that we have to know are the values of several ring diameters of the different lines and an approximate value of the etalon gap.

Let us suppose that two very close lines λ_a and λ_b (ν_a and ν_b) produce overlapping interference patterns, the corresponding diameters being D_{a1}, D_{a2}, $D_{a3}\cdots D_{b0}$, D_{b1}, $D_{b2}\cdots$. The central order numbers of λ_a and λ_b may be denoted by P_a and P_b. Then the fundamental formula (2.12) yields

$$P_a=p_a+\epsilon_a=2t\nu_a \quad \text{and} \quad P_b=p_b+\epsilon_b=2t\nu_b.$$

Thus, for the interval of the two lines we get:

$$\nu_a-\nu_b=(p_a-p_b)/2t+(\epsilon_a-\epsilon_b)/2t. \qquad (9.1)$$

The fractional parts ϵ_a and ϵ_b have to be calculated as described in (§3). The value of (p_a-p_b) will be a small integer or zero, but it remains undetermined until another photograph of the fringe system, taken with a different gap is at hand. From the change of the patterns then it is easy to determine whether the separation of the lines is smaller than the range of dispersion, or the orders overlap. In the first case, (p_a-p_b) is zero, in the second 1 or 2, or even more.

The evaluation remains the same if more than two lines form the interference system, but sometimes another procedure is advisable:

Using the expressions (2.7) and (2.11), for two consecutive rings of the a and b system we can write

$$\nu_a-\nu_b=(p_a-p_b)/2t + (D_{ak}^2/\Delta D_a^2-D_{bk}^2/\Delta D_b^2)/2t. \qquad (9.2)$$

As the lines are very close ΔD_a^2 and ΔD_b^2 will be the same ($=\Delta D^2=4f^2/\nu t$). Therefore,

$$\nu_a-\nu_b=(p_a-p_b)/2t+(D_{ak}^2-D_{bk}^2)/2t\Delta D^2. \qquad (9.3)$$

From this relation we see that not only the values $(D_{a(k-1)}^2-D_{ak}^2)$ and $(D_{b(k+1)}^2-D_{bk}^2)$ in the single ring systems are constant but also the corresponding values $(D_{ak}^2-D_{bk}^2)$ between two or more different systems. Thus, it is possible to arrange the observations of the D^2 in a rectangular scheme with constant horizontal (ΔH) and vertical (ΔV) differences. Sometimes, such a scheme will show, by discrepancies in the horizontal or vertical differences, that the one or the other D^2 value is far off and may be discarded.

The general scheme of an interference pattern, produced by three close components a, b, and c, can be arranged as follows:

ΔH	ΔH	ΔH	ΔH	ΔH		
D_{a0}^2	D_{a1}^2	D_{a2}^2	D_{a3}^2	D_{a4}^2	D_{a5}^2	
D_{b0}^2	D_{b1}^2	D_{b2}^2	D_{b3}^2	D_{b4}^2	D_{b5}^2	ΔV_1
D_{c0}^2	D_{c1}^2	D_{c2}^2	D_{c3}^2	D_{c4}^2	D_{c5}^2	ΔV_2.

Supposing that the orders do not overlap, the intervals $\nu_a-\nu_b$ and $\nu_b-\nu_c$ are given as

$$(\nu_a-\nu_b)=(D_{ak}^2-D_{bk}^2)/2t\Delta D^2=\langle\Delta V_1\rangle/2t\langle\Delta H\rangle,$$
$$(\nu_b-\nu_c)=(D_{bk}^2-D_{ck}^2/2t\Delta D^2=\Delta V_2/2t\langle\Delta H\rangle,$$

where the sign $\langle\rangle$ indicates the average values of the horizontal and vertical differences.

It should be mentioned that the intervals calculated in this way are not vacuum wave numbers, but corrections for vacuum are not necessary if we have to deal with lines of a small separation.

For the illustration of this method a special example may be given. It is taken from an earlier

work about isotopic shift in magnesium.[2] The patterns obtained with a 4.2-cm and a 3.6-cm etalon for the green line 5528A and with a 3.6-cm etalon for the infra-red line 8806A are reproduced in Fig. 9. The lines consists of three components which are approximately equally spaced. The strongest belongs to Mg^{24}, the weaker components to Mg^{25} and Mg^{26}. The measurement of the diameters and the calculation of their squares has yielded the D^2 values in mm² compiled in Table II in which the vertical and horizontal differences are also listed. From the values of the table, we take the average values:

$$\langle \Delta H \rangle = 43.8, \quad \langle \Delta V_1 \rangle = 11.2, \quad \langle \Delta V_2 \rangle = 11.0.$$

Therefore:

$$\Delta\nu_1 = 11.2/7.2016 \times 43.8 = 0.0355 \text{ cm}^{-1},$$
$$\Delta\nu_2 = 11.0/7.2016 \times 43.8 = 0.0349 \text{ cm}^{-1}.$$

The corresponding wave-length differences can be obtained from the relation

$$\Delta\lambda = -(\Delta\nu/\nu^2)10^8\text{A}.$$

If in a spectrum several close line groups of constant frequency differences are present (e.g., doublets or triplets of the sharp series), the equal spacing can be detected by a glimpse at a Fabry-Perot spectrogram.

As formula (9.2) shows, in the case of a doublet system of constant $(\nu_a - \nu_b)$ the expression $(D_{ak}^2 - D_{bk}^2)/\Delta D^2$ will be constant for all doublets. Using the fractional parts of the center orders P_a and P_b we can express this also by stating that $(\epsilon_a - \epsilon_b)$ will be the same for all doublets. Corresponding to this constant order difference the fringe system of one doublet component will be shifted relative to that of the other one in a characteristic manner. If the etalon gap is chosen so that a maximum separation of the two fringe systems will occur ("dissonance" for $\epsilon_a - \epsilon_b = 0.5$) this advantageous position will be obtained for all the doublets of constant frequency at the same time.

§10. Measurements of secondary wave-length standards

(*a*) *Determination of the exact value of the order p and of the etalon thickness t.*—As we saw, the determination of small wave-length differences or intervals is possible without knowledge of the exact value of the etalon gap. For precision measurements of secondary standard wavelengths, however, it is necessary to determine the precise value of the etalon thickness.

It is easy to find this if the interference patterns of at least two exactly known wavelengths are at our disposal and if an approximate value of t, say t', is known from measurements performed with a good screw micrometer or a comparator. An accuracy of ± 0.005 mm is sufficient.

Two methods are in use but they differ only with respect to the form of the expressions. The older method is based on the first form of the fundamental relation

$$p = 2t(1 - D^2/8f^2)/\lambda,$$

where p is the order of the ring with diameter D, t the thickness to be determined and f the focal length of the projecting system. D may be the diameter already reduced by means of the magnification factor of the spectrograph (§8).

The patterns of two lines λ_1 and λ_2 may yield the diameters D' and D'' of the innermost rings. Then we have

$$p_1 = 2t(1 - D''^2/8f^2)/\lambda_1$$

and

$$p_2 = 2t(1 - D''^2/8f^2)/\lambda_2.$$

Therefore, by dividing

$$p_2 = p_1 \cdot \lambda_1/\lambda_2 \cdot (1 - (D''^2 - D'^2)/8f^2). \quad (10.1)$$

The next step is now to determine the exact values of p_1 and p_2 for which only approximate values p_1' and p_2' can be obtained by the relations $p_1' = 2t'/\lambda_1$ and $p_2' = 2t'/\lambda_2$, t' being the approximate thickness. Considering that both p_1 and p_2 have to be whole numbers we put in the right side of the last equation (10.1) different integers $p_1, p_2, p_3 \cdots$ in the neighborhood of the approximate value p_1' and calculate in this manner a set of corresponding values $p_2', p_2'', p_2''' \cdots$. Only one of them will be so close to a whole number that the deviation may be due to unavoidable errors. This whole number will be p_2, and thus the unknown thickness can be found from the relation $2t = p_2\lambda_2/(1 - D''^2/8f^2)$.

It is clear that taking the $p_1^{(n)}$ over a wide range we will find several pairs $p_1^{(n)} p_2^{(n)}$ which

[2] K. W. Meissner, Ann. d. Physik **31**, 505 (1938).

will be integers. But only one of these will yield a value for the thickness t which will agree, within the limit of error, with the approximate value t'. A check of the result with more lines is always advisable.

Example.—The following data (squares of ring diameters in mm²) have been obtained from a photograph, taken with an etalon gap of the approximate thickness $t' = (12.054 \pm 0.002)$ mm and by means of a lens of focal length

$$f = (845.5 \pm 0.5) \text{ mm}, \quad f^2 \approx 715{,}000 \text{ mm}^2.$$

KRYPTON	D_5^2	D_4^2	D_3^2	D_2^2	D_1^2	D_0^2
5570.2890A	691.96	559.80	427.70	295.64	163.38	31.30
5870.9153A	712.84	574.03	434.72	296.32	156.78	18.02
5562.2251A	—	658.80	525.87	393.63	261.28	129.16

The approximate orders are calculated from the relation $p = 2t'/\lambda$:

$$p_1' = 43{,}280, \quad p_2' = 41{,}063, \quad p_3' = 43{,}342.$$

Table III contains the results of the calculations according to formula (10.1). Only the second rings (D_1^2) have been used. By inspection of this table, we see that the only possible values are: $p_1 = 43{,}277$, $p_2 = 41{,}061$, and $p_3 = 43{,}339$. Thus, the exact value of the thickness will be: $t = 12.0536141$ mm.

The newer method is based on the second form of the fundamental equation

$$P = 2t/\lambda = p + \epsilon,$$

where P is the (non-integer) order of the center of the interference system. It is equal to the order p of the first bright fringe plus the fractional part ϵ which, as we saw in §3, can easily be determined from the diameters of several interference circles. From the expression for ϵ

$$\epsilon = D_k^2 : [(D_k^2 - D_i^2)/(k-i)] - k$$

we see that the focal length of the projecting system need not be known, and that no reduction of the measured diameters need be made with respect to the magnification of the spectrograph. This gives a very important time-saving advantage to the method.

The determination of the thickness t by this method is very similar to the other described above. Let λ_1 and λ_2 be again the two known wave-lengths, $P_1 = p_1 + \epsilon_1$, and $P_2 = p_2 + \epsilon_2$ be their center orders, for which approximate values are obtained from the relation $P_1' = 2t'/\lambda_1$ and $P_2' = 2t'/\lambda_2$, t' being the approximate value of the etalon gap.

The correct values P_1 and P_2 have to satisfy the equation

$$P_1\lambda_1 = P_2\lambda_2 = 2t \quad \text{or} \quad P_2 = P_1\lambda_1/\lambda_2 \qquad (10.2)$$

or

$$(p_2 + \epsilon_2) = (p_1 + \epsilon_1)\lambda_1/\lambda_2. \qquad (10.3)$$

The correct values of P_1 and P_2 are now found in the following way. Using different values $(p_1' \pm i + \epsilon_1)$ in the neighborhood of $(p_1' + \epsilon_1)$, where p_1' and i are integers, we calculate, by the last relation, a corresponding set of values $(p_2' \pm k + \epsilon_k)$. The fractional ϵ_k of these values generally will be different from ϵ_2, but one of them, let us say ϵ_m, will be very close to the measured ϵ_2, so that the equation

$$(p_1' + n + \epsilon) = (p_2' + m + \epsilon_2)\lambda_2/\lambda_1$$

will hold. If at the same time the value $(p_1' + n + \epsilon)$ equals $2t'$, within the limit of error of this preliminary thickness value, we can make the conclusion

$$p_1 = p_1' + n \quad \text{and} \quad p_2 = p_2' + m.$$

A check with a third or fourth known standard line is advisable. It is convenient to employ three lines, λ_1, λ_2, λ_3, for which $(\lambda_1 - \lambda_2)$ is not too small, and $(\lambda_1 - \lambda_3)$ is small. The red neon lines, or the green and yellow krypton lines are very convenient for this purpose.

Example.—The same data we used for the illustration of the first method may be applied to elucidate the second method. The fractional parts of the three krypton lines are found from these data as $\epsilon_1 = 0.237$, $\epsilon_2 = 0.130$, and $\epsilon_3 = 0.980$. The results of the calculations according to formula (10.3) are compiled in Table IV. The inspection of this table shows that the only possible value of P_1 can be 43,278.237. The exact thickness of the etalon gap, calculated with this value, is found as $t = 12.053614$ mm, in accordance with the result calculated with the first method.

If three standard lines are available, it is sometimes advisable to calculate the possible range of the orders in the following manner: Using the same designations as before, we find from the relations

$$(p_1 + \epsilon_1)\lambda_1 = (p_2 + \epsilon_2)\lambda_2 = (p_3 + \epsilon_3)\lambda_3$$

TABLE III.

p_1	p_2	p_3
43,238.000	41,024.000	43,299.945
239.000	024.949	300.946
43,257.000	41,042.027	43,318.972
258.000	042.976	319.973
43,276.000	41,060.055	43,338.000
277.000	061.003	339.001
278.000	061.952	340.002
43,296.000	41,079.030	43,358.028
297.000	079.979	359.029
43,315.000	41,097.058	43,377.055
316.000	098.006	378.056

the following expressions for the central order of the line λ_1:

$$(p_1+\epsilon_1)=(p_1-p_2+\epsilon_1-\epsilon_2)\lambda_2/(\lambda_2-\lambda_1)$$

and

$$(p_1+\epsilon_1)=(p_1-p_3+\epsilon_1-\epsilon_3)\lambda_3/(\lambda_3-\lambda_1).$$

All items of the right sides are known, except (p_1-p_2) and (p_1-p_3) which are known only within a few units from the approximate values p_1', p_2' and p_3'. By substitution of a few values for the order differences, the size of which differ only by a few units from the approximate values $(p_1'-p_2')$ and $(p_1'-p_3')$, we calculate two different sets p_1'' and p_1''' from the two last equations. Two values of these sets will be very close and correspond to the correct p_1 value. After this preliminary determination one of the earlier described methods has to be applied. A simple error calculation will show that this method cannot be applied for the exact determination of p_1 or for the checking of the fractional parts.

This preliminary method may be illustrated with the same data we used above. From the approximate values p_1', p_2' and p_3' (see other examples) we find the approximate differences $(p_1-p_2)=2217$ and $(p_3-p_1)=63$. With these values we find the following two sets of p_1'' and p_1''' values:

p_1-p_2	$P_1''=p_1''+\epsilon_1$	p_3-p_1	$P_1'''=p_1''+\epsilon_1$
		61.743	42,588.383
2215.107	43,258.707		
2216.107	43,278.236	62.743	43,278.151
2217.107	43,297.765		
		63.743	43,967.920

As we see from these values, the only possible value of p_1 in a range of thickness of about 0.2 mm is $p_1=43,278$, the same value we found with the other methods. (In the first sample we found 43,277 for the second ring, so that the first ring has to have the order 43,278.)

(*b*) *Measurement of secondary wave-length standards.*—The primary standard for wave-length measurements is the red cadmium line, adopted as 6438.4696A in dry air at 76 cm Hg atmospheric pressure and at 15°C. A reliable comparison of other wave-lengths with this standard line is possible only by means of interferometric measurements. A large number of iron, neon and krypton lines, distributed over the whole range of the spectrum at an average distance of 50A, are determined in this manner and are called secondary standards. If the definition of the lines is good, it is possible to obtain the accuracy of a few ten thousandths A, with which the primary standard is known with respect to the international meter.

The cadmium spectrum is produced by a vacuum tube of the same type as used originally by Michelson and later by Fabry and Benoît. By means of a half-silvered mirror, the standard light source and the source the spectrum of which is to be studied, are projected simultaneously on the proper part of the interferometer arrangement. Thus the patterns of the standard line and unknown lines are affected by slight changes of temperature or pressure in the same amount and discrepancies may be avoided. If the intensities of standard line and secondary line are too different, the stronger source should be weakened by a neutral filter of proper absorption power.

TABLE IV.

5570.2890A $P_1=p_1+\epsilon_1$	5870.9153A $P_2=p_2+\epsilon_2$	5562.2251A $P_3=p_3+\epsilon_3$	DOUBLE THICKNESS $2l$
43,239.237	41,025.127	43,301.921	24.0855 mm
240.237	026.076	302.922	
43,258.237	41,043.154	43,319.947	24.0961 mm
259.237	044.103	320.948	
43,277.237	41,061.181	43,339.976	
278.237	062.130	340.977	24.1072 mm
279.237	063.078	341.978	
43,297.237	41,080.156	43,360.005	24.1178 mm
298.237	081.104	361.006	

For high precision measurements several photographs of the spectra have to be taken with different, at least two, etalon gaps. For the evaluation of the diameter measurements of the ring systems, we may use one of the two methods described above.

The older method is based on the formula (2.6) in which we denote the items referring to the standard line by an index, s, and the items of the unknown wave-lengths by an index, x, so that we have

$$p_x = p_s(\lambda_s/\lambda_x)(1-(D_x^2-D_s^2)/8f^2)$$

or

$$\lambda_x = \lambda_s(p_s/p_x)(1-(D_x^2-D_s^2)/8f^2). \quad (10.4)$$

For a given diameter D_s, already reduced with respect to the magnification of the spectrograph, the order p_s is to be determined in the manner described under (a). The only unknown of the last equation are now λ_x and p_x. From grating measurements, the unknown wave-length usually is known with an accuracy of at least 0.1A and this is sufficient for the determination of an approximate value p_x' of p_x according to (2.6). Preferably for small etalon gaps, p_x' will then be rather close to a whole number, the p_x in question. By putting this integer into the right side of Eq. (10.4), we obtain the correct value of λ_x. However, this value is not the final precise value; two more corrections have to be applied which are explained in §11 and §12.

It is perhaps useful to write down Eq. (10.4) so that all items appear which have to be determined. If d_x and d_s are the diameters taken from the photograph and if k_x and k_s are the magnification factors for both lines, the equation is:

$$\lambda_x = \lambda_s(p_s/p_x)(1-d_x^2/k_x^2f^2-d_s^2/k_s^2f^2).$$

This form shows that a great many annoying calculations are involved. In order to avoid unnecessary calculations and incorrect averaging it is advisable to derive first from the different measured d values of a line the average value d_0^2 of the first circle and to use this value for the further treatment.

Very much simpler is the method developed by St. John using the orders of the ring centers $P=p+\epsilon$. In this case, the formula for the connection of standard line and unknown line becomes:

$$(p_x+\epsilon_x) = (p_s+\epsilon_s)\lambda_s/\lambda_x$$

and

$$\lambda_x = \lambda_s(p_s+\epsilon_s)/(p_x+\epsilon_x).$$

The calculation of the fractional part ϵ is rather simple, as described in §3. The value of p_s is to be determined with the aid of a second exactly known wave-length according to §10(a). An approximate value of p_x can be found if λ_x is known within 0.1A from grating measurements. Thus the exact integer p_x can be determined, which, in connection with the known fractional part ϵ_x yields the unknown wave-length λ_x.

This formula is very convenient for the use of a calculating machine. For the evaluation of a whole spectrogram, the numerator $\lambda_s(p_s+\epsilon_s)$ remains constant and thus the last calculations are quite a pleasure if the approximate values p_x for the different lines are determined beforehand.

§11. Correction for phase change at reflection

If we determine the wave-length λ_x with respect to the standard line λ_s in the manner described in the last part, we find that the results, obtained with different etalon gaps, vary systematically with increasing thickness t. This may be illustrated by the following example in which the infra-red caesium line 8078.9A has been measured with respect to the caesium line 6354.552A as standard. (This value has been obtained by direct comparison with the red Cd line.)

Gap	6.0 mm	10.0 mm	16.0 mm
Wave-length	8078.931A	8078.928A	8078.925A

This behavior can only be understood if we assume that the fundamental formula (2.12), $P=2t/\lambda$, is not quite correct. Indeed, deriving this formula, we did not take into account the phase change which the wave trains have to undergo when reflected at the surface of the metal film of the interferometer plates. Introducing a phase change ψ for each reflection at S_1 and S_2 (Fig. 2), we get for the order number p an additive term $2\psi/2\pi=\delta$, so that the directions corresponding to the integers p will differ from the original ones. As the equations show, we can describe the influence of the phase change also by the assumption of an "effective distance" $t'=(t+\tau)$ which is different from the geometrical

distance. In all our calculations, therefore, the original t has to be replaced by the effective thickness. But no systematical changes of the wave-lengths arise as long as τ is a constant for all wave-lengths. However, as soon as we assume that there is a *dispersion of phase change*, or, in other words, that τ is a function of λ, we can expect such a systematical drift of the calculated wave-length λ_x to occur with increasing gap (increasing order). The effective thickness for two lines λ_s and λ_x may be $(t+\tau_s)$ and $(t+\tau_x)$ so that the slight change in thickness will be $(\tau_s-\tau_x)$. Then the error we make by calculating the wave-length without regarding the phase change is given by $2(\tau_s-\tau_x)/P_x$, a value which, indeed, decreases with increasing order.

For exact comparisons of wave-lengths by means of the Fabry-Perot interferometer, it is necessary to eliminate the effect of dispersion of phase change, for it produces systematic errors which, sometimes, are many times greater than the accidental errors of one measurement. It can be done if at least two different gaps t_1 and t_2 are used. The formulae for the elimination or determination of the phase change first may be derived as follows:

Introducing the effective thickness $(t_1+\tau_s)$ and $(t_1+\tau_x)$ for the standard and unknown lines (λ_s and λ_x), the center orders of which may be P_s and P_x, we write the fundamental formula for two thicknesses t_1 and t_2:

$$\begin{aligned} P_s' &= 2t_1/\lambda_s + 2\tau_s/\lambda_s, \\ P_s'' &= 2t_2/\lambda_s + 2\tau_s/\lambda_s, \\ P_x' &= 2t_1/\lambda_x + 2\tau_x/\lambda_x, \\ P_x'' &= 2t_2/\lambda_x + 2\tau_x/\lambda_x. \end{aligned} \qquad (11.1)$$

The simplest way to eliminate the phase change is the following: By subtracting the equations for standard and unknown lines from each other we find two new equations from which, by division, the unknown λ_x is found in terms of λ_s and the observed orders P. The result is:

$$\lambda_x = \lambda_s(P_s''-P_s')/(P_x''-P_x'). \qquad (11.2)$$

The P values are determined in one of the manners described in (§10 (*a*)).

This method is simple and convenient, when only a few lines have to be measured. In a comprehensive work, however, when a great many lines have to be compared with a standard line, usually another way is chosen which allows a check of the consistency of the single measurements. In this method one determines either the difference of the effective thickness (and therewith the phase change correction) for the unknown lines with respect to the standard line or directly the corrections, which have to be added to the wave-lengths obtained without regarding the phase effect.

The corresponding relations can be derived from the quadruple equations (11.1). By dividing the equations belonging to t_1, we get

$$\begin{aligned} P_s'/P_x' &= (\lambda_x/\lambda_s)(t_1+\tau_s)/(t_1+\tau_x) \\ &= (\lambda_x/\lambda_s)(1+(\tau_s-\tau_x)/t_1) \end{aligned}$$

or

$$\begin{aligned} \lambda_x &= (\lambda_s P_s'/P_x')(1+(\tau_x-\tau_s)/t_1) \\ &= \lambda_x' + \lambda_x'(\tau_x-\tau_s)/t_1 = \lambda_x' + \Delta\lambda_x' \end{aligned} \qquad (11.3)$$

where λ_x' is the wave-length we obtained by the methods described in §10 (*b*). For the etalon thickness t_2, we find in the same manner:

$$\begin{aligned} \lambda_x &= (\lambda_s P_s''/P_x'')(1+(\tau_x-\tau_s)/t_2) \\ &= \lambda_x'' + \lambda_x''(\tau_x-\tau_s)/t_2 = \lambda_x'' + \Delta\lambda_x''. \end{aligned}$$

From the last two equations we find the phase change correction δ in order numbers as

$$\delta = \frac{2(\tau_x-\tau_s)}{\lambda_x} = \frac{\lambda_x''-\lambda_x'}{\lambda_x^2}\cdot\frac{2t_1t_2}{t_2-t_1} \qquad (11.4)$$

and the wave-length corrections for both gaps as

$$\Delta\lambda_x' = (\lambda_x''-\lambda_x')t_2/(t_2-t_1)$$

and

$$\Delta\lambda_x'' = (\lambda_x''-\lambda_x')t_1/(t_2-t_1). \qquad (11.5)$$

The last two values have to be added to the uncorrected wave-lengths λ_x' and λ_x'', obtained form the gaps t_1 and t_2, respectively. These corrections are different for the different gaps and proportional to $1/t$ for a given line. A graph of the corrections obtained for different lines may be made for each distance and for the final corrections the average corrections may be used.

The expression for the phase change correction δ does not depend upon the gap size. Therefore, *one* graph of δ against the wave-length can contain all the values obtained by the combination of the different gaps. This fact allows one to check the accuracy of the single wave-length values obtained in a long series of measurements in

TABLE V.

COMBINATION GAP	6 MM AND 10 MM	10 MM AND 16 MM	6 MM AND 16 MM	AVERAGE	WAVE-LENGTH UNCORRECTED	WAVE-LENGTH CORRECTED
6 mm	−0.008		−0.010	−0.009	8078.931	8078.9220
10 mm	−0.005	−0.008		−0.0065	.928	.9215
16 mm		−0.005	−0.004	−0.0045	.925	.9205
					Average:	8078.921A ±0.001A

secondary standard work. Sometimes it is advisable to employ the average values of the phase correction for a recalculation of the whole material. The first part of formula (11.3) then conveniently is transformed in the following way:

$$\lambda_x=\lambda_s P_s'/P_x'(1-(\tau_x-\tau_s)/l_1)$$
$$=\lambda_s P_s'/(P_x'-2(\tau_x-\tau_s)/\lambda_x)$$
$$=\lambda_s P_s'/(P_x'-\delta). \quad (11.6)$$

It is self-evident that the knowledge of the phase change correction enables us to make interferometric measurements by means of one etalon gap. For a given pair of metal coated plates the optical properties of the reflecting surfaces usually remain unchanged several years and so the phase change correction stays constant. But for each new coating it must be determined anew.

Before we add some remarks, we will apply the correction formula for $\Delta\lambda_x'$ to the example of the caesium line 8078.9A, which was used to show the systematic drift of the wave-length values with increasing gap. In Table V the different corrections $\Delta\lambda_x'$ and $\Delta\lambda_x''$, obtained by combining the values of each two distances are compiled. By adding the averaged corrections to the uncorrected λ_x values the corrected wave-length is obtained. The average of the phase change correction δ is $+0.020$, as can easily be checked.

Introducing the difference of the effective thickness of the interferometer for standard and unknown line, $2(\tau_x-\tau_s)=\delta\cdot\lambda_x$, we find in this case that $2(\tau_x-\tau_s)=16$ mμ. That means that the effective thickness for 8079A is greater than that for 6354A by 8 mμ.

It is clear that the phenomenon called "dispersion of phase change" is not explained by the rough hypothesis of an "effective thickness" or by the assumption that at the reflection from metallic films, the light penetrates the film somewhat and that the depth of penetration is a function of the wave-length. However, it is possible to explain the phenomenon by the application of the electromagnetic theory of metals. At the suggestion of the author, Juergen Bauer took up this problem, and was able to show that the phase change correction can be derived from the optical constants and the thickness of the metal film. The shape of the curves he calculated for different metals corresponds in every regard to the experimental curves. For our purpose, it is unnecessary to go into detail; only one remarkable result of Bauer's investigation may be stated, namely that, by combining different metals, it must be possible to produce a film for which the dispersion of the phase change is very small.

It is self-evident that the correction for phase change is only necessary if measurements of wave-lengths at a larger distance from the standard line have to be made. In all measurements of fine or hyperfine structure it does not play any role.

It may be mentioned that the phase change correction can be determined also with interferences of equal thickness, formed at the surface of a wedge-shaped film. By putting together the interferometer plates in such a way that the metal coated surfaces formed a very thin air wedge, Fabry and Buisson determined the dispersion of phase change for several metal films deposited on glass or quartz by sputtering or chemically.

However, according to our own experience, the method described above, using the combination of etalons of different thickness should be preferred. The application of the wedge interferences affords an appreciable change of the set-up because the interference fringes are situated at the surface of the film so that the projecting system has to be moved if one projects the fringes on the slit of the spectral apparatus. But

more important is the fact that by this method the phase change is found only for such areas of the plates as are focused on the slit. Thus, slight inhomogeneities of the film qualities may influence the results. The method of small and large etalons yields an average value of the phase change correction for the total area used for the wave-length determination.

§12. Correction of wave-length measurements for the dispersion in air

The wave-length values obtained after the elimination of the influence of phase change usually have to be reduced to their values under normal conditions, which are defined by dry air at 15°C and a barometric pressure of 76 cm Hg. Although the absolute values of the wave-lengths are affected appreciably by the variation of the air conditions, the relative values are changed only slightly. If the index of refraction were a constant for given air conditions, the variation of all wave-lengths would be in the same ratio and the correction to be applied would be simple. The dispersion of air, however, causes the variation to be a function of λ, so that the necessary corrections have to be calculated for each wave-length. The corrections are small but cannot be neglected in accurate measurements.

The correction to be added is

$$\Delta=\lambda(n_0-n_0')\times(\rho-\rho_0)/\rho_0,$$

where λ is the wave-length of the unknown line, n_0 the refractive index for this wave-length, n_0' that of the primary standard, both at normal conditions, ρ the density of the air for the conditions (t°C, h cm Hg) at which the measurements are made, and ρ_0 the density for standard condition (15°C, 76 cm Hg).

The factor $(\rho-\rho_0)/\rho_0=f(t, h)$ can easily be calculated and tabulated. It is constant for all lines of the same spectrogram. Also Δ can be tabulated as a function of $(\rho-\rho_0)/\rho_0$ and λ so that the correction Δ can very easily be obtained. Convenient tables have been calculated by Meggers and Peters[3] for Cd 6438A as primary standard.

These tables may be used also in the case when another line is chosen as a standard. Let us suppose λ'' and n_0'' may be the wave-length and refractive index of this line. The correction we want is

$$\Delta'=\lambda(n_0-n_0'')\times(\rho-\rho_0)/\rho_0.$$

The tables give the values

$$\Delta=\lambda(n_0-n_0')\times(\rho-\rho_0)/\rho_0$$

and

$$\Delta''=\lambda''(n_0''-n_0')\times(\rho-\rho_0)/\rho_0,$$

from which we get

$$\Delta'=\Delta-\Delta''-((\lambda-\lambda'')/\lambda'')\times\Delta''.$$

Derivation of the correction formula.—Let λ, λ', n and n' represent wave-lengths and refractive index for these wave-lengths at t°C and 76 cm Hg where the prime refers to the standard line and the unprimed letters to the unknown line; let λ_0, λ_0', n_0, and n_0' further be the same quantities at standard conditions. Then we have, using p and p' as order numbers

$$\lambda=p'\lambda'/p.$$

But in the calculations, λ_0' was used instead of λ' and the relation was

$$\lambda=p'\lambda_0'/p.$$

The difference between the exact value λ_0 and that calculated is the correction we want:

$$\Delta=\lambda_0-p'\lambda_0'/p=\lambda_0(1-p'\lambda_0'/p\lambda_0)$$
$$=\lambda_0(1-\lambda\lambda_0'/\lambda'\lambda_0).$$

But $\lambda/\lambda_0=n_0/n$ and $\lambda'/\lambda_0'=n_0'/n'$, therefore

$$\Delta=\lambda_0(1-n_0n'/n_0'n)=\lambda_0(n_0'n-n_0n')/n_0'n$$
$$=\lambda_0(n_0'n-n_0n'),$$

for the product $n_0'n$ is approximately equal to one. This formula can be transformed in the final form by applying the known relation between refractive index and density (index 0 refers to standard conditions):

$$(n-1)/\rho=(n_0-1)/\rho_0$$

and

$$(n'-1)/\rho=(n_0'-1)/\rho_0.$$

The values of n and n' resulting from these relations are introduced in the last expression for the correction, which finally has the form:

$$\Delta=\lambda_0(n_0-n_0')(\rho-\rho_0)/\rho_0.$$

It is clear that λ_0 can be replaced by λ.

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Thermal Conductivities of Tungsten and Molybdenum at Incandescent Temperatures

ROBERT H. OSBORN*
Physics Department, University of Pittsburgh, Pittsburgh, Pennsylvania
(Received April 7, 1941)

Using an optical pyrometer method, the thermal conductivity of tungsten has been found to vary linearly from 1.170 watts/(cm² K°/cm) at 1100°K to 1.026 watts/(cm² K°/cm) at 2000°K. Similarly, the thermal conductivity of molybdenum has been found to vary linearly from 1.083 watts/(cm² K°/cm) at 1200°K to 0.666 watt/(cm² K°/cm) at 1900°K. Spectrographic analysis indicated only traces of impurities in samples of the two metals used in the investigation.

I. INTRODUCTION

THE thermal conductivities of tungsten, tantalum and carbon at incandescent temperatures have been measured by Worthing[1] in the range from 1500°K to 2500°K by an optical pyrometer method. Filaments drawn from the three elements were mounted in evacuated bulbs, and temperature gradients were measured along each filament near its junctions with the current leads into the bulb. These data, together with data on the radiancies $\mathfrak{R}$ of the elements as functions of absolute temperature T were used to compute thermal conductivities k according to the relationship:

$$k=\frac{2\mathfrak{R}_{\max}}{rdT/dl}\int_l^L\left(\frac{\omega}{\omega_{\max}}-\frac{\mathfrak{R}}{\mathfrak{R}_{\max}}\right)dl, \qquad (1)$$

where r is the radius of the filament, l is the distance from the cooling junction measured along the filament, and ω is the electrical power input at a given filament cross section per unit of radiating surface.

Due to the fact that it was impossible to bring those portions of the filaments near the cooling junctions to a temperature sufficiently high for proper aging, temperature gradient readings were of necessity made on poorly aged regions. Also, at that time, the radiation characteristics of the materials used had not been accurately determined. For these reasons, it was decided that a redetermination of the thermal conductivity of tungsten would be desirable due to the commercial importance of the metal. At the same time, it was decided to extend the method to molybdenum.

II. EXPERIMENTAL PROCEDURE

The experimental arrangement used in the present work consisted of a Pyrex glass bulb in which were mounted the various filaments and which was sealed to a vacuum system consisting of a mercury condensation pump backed by a Cenco Hy-Vac oil pump. Condensible vapors were removed by a liquid-air trap. To prevent vibration while measurements were being taken on the unsupported hairpin filaments, the entire vacuum system, with the exception of the oil pump, was suspended by wide strips of live rubber from a steel framework. The filaments were of 10- or 20-mil wire polished with 4-0 emery paper and powdered alumina. Micrometer measurements along the length showed that the diameter of each filament was uniform to within 2 or 3 parts in a thousand. Spectrographic analyses of both the tungsten and molybdenum samples showed that they contained only traces of metallic impurities.

Aging of the filaments was carried on in a separate bulb. The tungsten filaments were aged for two hours at approximately 2700°K, while the molybdenum filaments were aged at about 2200°K for fifteen minutes.

After aging, the average diameters of the filaments were determined by means of published values of density[2] and measurements of mass and length. Each filament was then mounted in turn

* Now at Hercules Experiment Station, Hercules Powder Company, Wilmington, Delaware.

[1] A. G. Worthing, Phys. Rev. **4**, 535 (1914).

[2] W. P. Davey, Phys. Rev. **25**, 753 (1925).

in the Pyrex bulb in the manner shown in Fig. 1. Potential leads of 0.7-mil tungsten were tied to the filament at measured distances from the ends. A central electrode of 40-mil tungsten to which was applied a negative potential of 90 volts with respect to the filament was used to stop electron emission during subsequent measurements. The cooling junction and spring shown in Fig. 1 were absent during the following calibration process.

With a given filament in position, the bulb was evacuated and readings were taken of current through the filament, voltage between the potential leads, and temperature of the filament below the potential lead junctions for several more or less equally spaced values of current such that the maximum temperatures below the potential lead junctions varied from 1200°K to 2000°K, approximately. The cooling junctions with the current leads were far enough removed from the potential lead junctions so that for all currents used, the temperatures of all points below the potential leads were essentially the same for a given value of current.

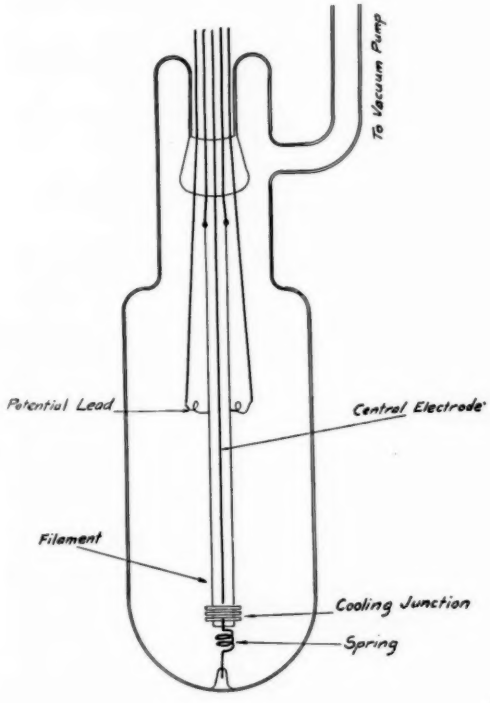

FIG. 1. Design of vacuum tube.

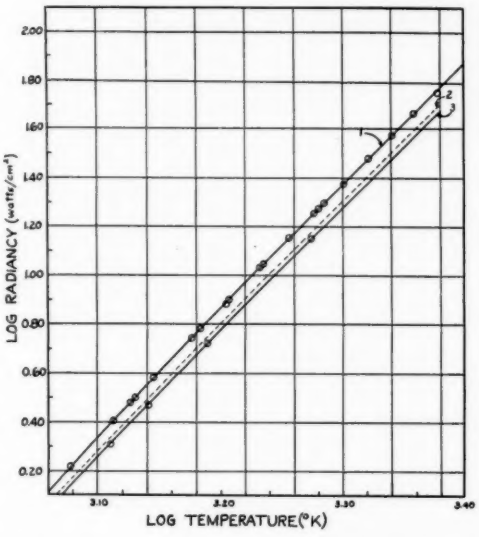

FIG. 2. Radiancy-temperature relationships for tungsten and molybdenum. Curve 1—relationship for tungsten found by Forsythe; points determined by author. Curve 2—relationship for molybdenum given by Worthing. Curve 3—relationship for molybdenum determined by author.

Current and voltage measurements were made with a Leeds and Northrup Type K potentiometer, and temperature readings were taken with a disappearing filament optical pyrometer which had been calibrated carefully over a range of brightness temperatures from 1000°K to 2700°K. The pyrometer was rigidly mounted on a cathetometer standard in place of the original telescope. In this manner, temperature readings could be taken on portions of the filament at measured distances from any fixed point.

The current-voltage-temperature data, together with measurements of the average diameter of the filament and the length of filament between potential leads served to determine the radiancy $\mathfrak{R}$ and the resistivity ρ as functions of T. Figure 2 is a plot of log radiancy *vs.* log temperature for typical samples of well-polished tungsten and molybdenum. Figure 3 is a plot of resistivity *vs.* temperature for the same samples. Values of both properties obtained for tungsten are in close agreement with corresponding values pub-

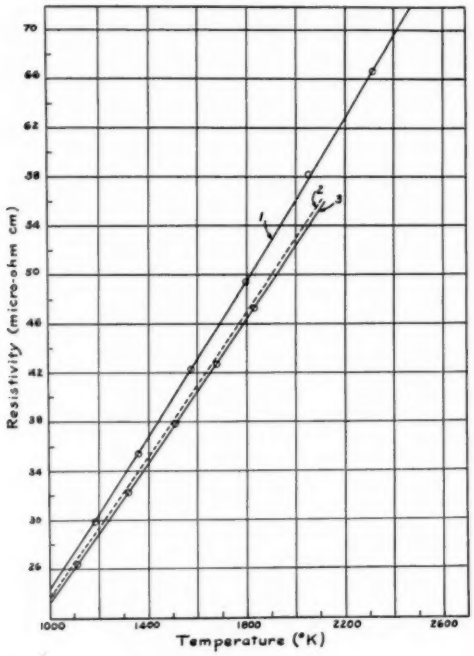

FIG. 3. Resistivity-temperature relationships for tungsten and molybdenum. Curve 1—relationship for tungsten found by Forsythe; points determined by author. Curve 2—relationship for molybdenum given by Worthing. Curve 3—relationship for molybdenum determined by author.

lished by Forsythe,[3] while those obtained for molybdenum are somewhat lower in each case than those given by Worthing.[4] The purity of the molybdenum wire used by the author may have been greater than the purity of the wire measured by Worthing, which would account for the lower resistivities found in the present work.

Next, the tube was opened and three pieces of 85-mil nickel wire were welded in an atmosphere of hydrogen between the two legs of the filament near its central portion. A spring of 12-mil tungsten wire, attached to the lower piece of nickel, helped to eliminate filament vibration during the next series of measurements. This arrangement is shown diagrammatically in Fig. 1. At this point the potential leads were removed. The tube was again evacuated, and a series of readings was taken of the surface temperatures along one leg of the filament near the nickel cooling junction for each of two or three maximum temperatures. At each temperature, by averaging readings taken with current flowing first in one direction and then in the other, the effects of the Thompson electromotive force were eliminated. Thus T as a function of l was determined. Figure 4 shows six typical temperature distribution curves which were plotted from data obtained in the foregoing manner.

III. Supplementary Procedures

The standard of brightness temperature was a hairpin filament tungsten lamp which was calibrated at the melting point of palladium (taken as 1829°K) by Dr. W. E. Forsythe of the Lamp Development Laboratories of the General Electric Company, Nela Park.

Limiting effective wave-lengths of the red glass filters used with the disappearing filament optical pyrometer were also furnished by Dr. Forsythe.

The temperature scale was based on Wien's equation where the constant C_2 was taken as $14{,}320\mu K°$.

True temperatures for tungsten and molybdenum were obtained from brightness temperatures by means of Wien's law and the values of spectral emissivities for the two metals given by Worthing.[5]

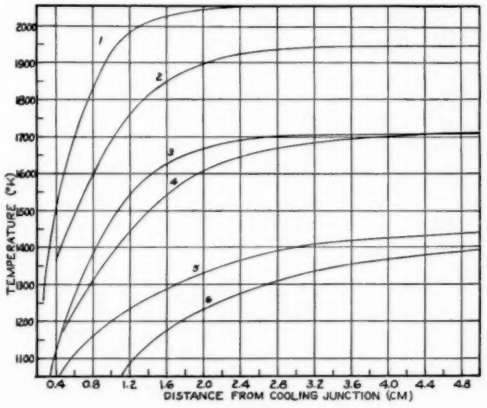

FIG. 4. Temperature distribution curves. Curve 1—10-mil tungsten; $T_{max}=2054°K$. Curve 2—20-mil molybdenum; $T_{max}=1947°K$. Curve 3—10-mil tungsten; $T_{max}=1703°K$. Curve 4—20-mil molybdenum; $T_{max}=1709°K$. Curve 5—20-mil tungsten; $T_{max}=1440°K$. Curve 6—20-mil molybdenum; $T_{max}=1404°K$.

[3] W. E. Forsythe, J. Opt. Soc. Am. **24**, 114 (1934).
[4] A. G. Worthing, Phys. Rev. **25**, 846 (1925).
[5] A. G. Worthing, Phys. Rev. **10**, 377 (1917); **25**, 846 (1925).

Frequent measurements of bulb transmissions were made by the usual method of measuring the brightness of an image of an auxiliary lamp filament both with and without the bulb in the light path from the filament to the pyrometer.

IV. Calculations

The remaining relations needed for determining thermal conductivities were those expressing ω/ω_{max} and $\mathfrak{R}/\mathfrak{R}_{max}$ as functions of the distance from the cooling junction. ω/ω_{max} is given by the relation:

$$\frac{\omega}{\omega_{max}}=\frac{\rho}{\rho_{max}}. \tag{2}$$

Since ρ as a function of temperature is known (Fig. 3), ω/ω_{max} as a function of l may be found from Eq. (2) and the curves of Fig. 4. The relations between $\mathfrak{R}/\mathfrak{R}_{max}$ and l were found by means of the previously determined relation between $\mathfrak{R}$ and temperature (Fig. 2), and the curves of Fig. 4.

Worthing assumed the following empirical expressions for his experimental values:

$$\mathfrak{R}=\sigma T^{\beta}, \tag{3}$$

or

$$\log \mathfrak{R}=\log \sigma+\beta \log T, \tag{3a}$$

and

$$T=T_{max}[1-\exp[-\mu(l+l_0)]]^{\gamma}, \tag{4}$$

TABLE I. *Effect of temperature on thermal conductivities and Lorenz numbers of tungsten and molybdenum.*

	True Temperature (°K)	Thermal Conductivity Watts/(cm² K°/cm)	$\frac{k}{xT}$ (Micro-Ohm Watts/K°²)
Tungsten			
	1100	1.170	0.0290
	1200	1.153	0.0292
	1300	1.138	0.0293
	1400	1.122	0.0293
	1500	1.106	0.0293
	1600	1.089	0.0292
	1700	1.073	0.0291
	1800	1.058	0.0290
	1900	1.042	0.0289
	2000	1.026	0.0287
Molybdenum			
	1200	1.083	0.0261
	1300	1.023	0.0250
	1400	0.964	0.0238
	1500	0.904	0.0226
	1600	0.844	0.0213
	1700	0.785	0.0200
	1800	0.725	0.0186
	1900	0.666	0.0173

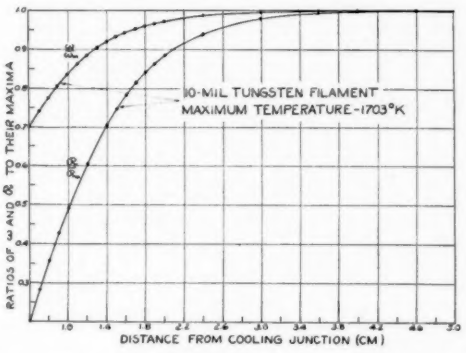

FIG. 5. Curves of the type from which the integrals of Eq. (1) were obtained.

where σ, β, μ, l_0 and γ are constants. Equation (3a) indicates a straight line relationship between log radiancy and log temperature which does not strictly hold as may be seen from an inspection of Fig. 2. Equation (4) describes the temperature distribution in the neighborhood of the cooling junction. An attempt was made to fit the data of the present work into this equation, but the deviations encountered in the best possible fit were considered greater than those which might be attributed to experimental error. Hence, derivatives dT/dl were found graphically, and the integrals

$$\int_l^L \frac{\omega}{\omega_{max}}dl \quad \text{and} \quad \int_l^L \frac{\mathfrak{R}}{\mathfrak{R}_{max}}dl$$

were obtained by a summation method from curves such as those in Fig. 5. Thus, all the data necessary for calculating k from Eq. (1) were at hand.

V. Results

The values for the thermal conductivities of tungsten and molybdenum computed from Eq. (1) are given in Table I. Values of the Lorenz number k/xT, where x is the electrical conductivity, have been included in the data. This quantity shows a much greater variation in the case of molybdenum than in the case of tungsten. In Fig. 6 are shown plots of thermal conductivity *vs.* temperature for the two metals. If the curve for tungsten is extrapolated back to room temperature, a value of 1.3 watts/(cm² K°/cm) is obtained. Various investigators have reported val-

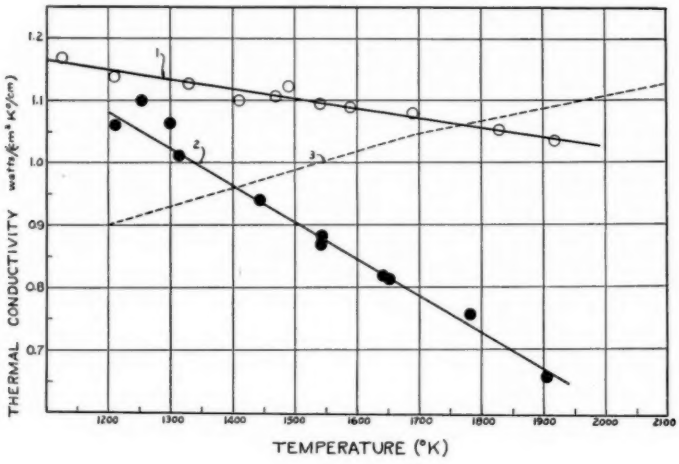

FIG. 6. Thermal conductivity *vs.* temperature. Curve 1—tungsten. Curve 2—molybdenum. Curve 3—Worthing's relationship for tungsten.

ues in the neighborhood of 1.6 watts/(cm² K°/cm) for tungsten at room temperature. Extrapolation of the molybdenum curve yields 1.6 watts/(cm² K°/cm) compared with a value of 1.46 watts/(cm² K°/cm) given by Weber.[6] Results published by Worthing[7] for the thermal conductivity of tungsten are plotted in Fig. 6 for comparison. These values were recomputed from the data which he obtained in 1914 on the basis of a later temperature scale for tungsten.[8] The break in the curve was not present in the 1914 publication and its presence cannot be explained as a result of the change in the temperature scale. Due to the unavailability of his original data, it is impossible to make any conjectures regarding its cause.

[6] S. Weber, Ann. der Physik **54**, 165 (1917).
[7] A. G. Worthing, Astrophys. J. **61**, 52 (1925).
[8] A. G. Worthing, Phys. Rev. **10**, 377 (1917).

VI. SUMMARY

The thermal conductivities of tungsten and molybdenum have been measured in the range from 1100°K to 2000°K by an optical pyrometer method. A linear relationship between thermal conductivity and temperature was found for each metal, with values of thermal conductivity decreasing with increasing temperatures. Extrapolations to room temperature yield values in reasonable agreement with previously published measurements.

Resistivities and radiancies found for tungsten are in good agreement with values given by Forsythe, while those found for molybdenum are somewhat lower than those given by Worthing.

It is a pleasure to acknowledge the cooperation of Dr. A. G. Worthing who suggested many of the improvements in the experimental procedure.

Properties and Use of a High Frequency Spark Discharge for Local Micro-Analysis

WILLIAM M. MURRAY, JR., BRIGGS GETTYS AND SAMUEL E. Q. ASHLEY
General Electric Company, Pittsfield Works' Laboratory, Pittsfield, Massachusetts

(Received April 3, 1941)

THE present paper deals with the properties of a highly localized spark used with a spectrograph for the examination of the composition of restricted minute areas.

The fundamental idea of the use of the spectrograph for the examination of very small areas of a specimen is not new. Many investigators have used small sparks, arcs, and Tesla discharges in conjunction with a moving spectrographic plate to study segregation.

The present investigation covers a study of the type of electrical discharge produced and the spectra obtained by the use of the exciting source for local micro-analysis which was described in a generalized form by G. Scheibe,[1] again by G. Scheibe and J. Martin,[2] and by A. Henrici and G. Scheibe.[3] A thesis containing a detailed report of the development and results of the early work was published by J. Martin.[4] The intention of this new technique is to use a direct current high voltage spark which is guided over a moving specimen while the spectrographic plate on which the spectrum is being recorded is moved vertically. This results in a continuously changing spectrogram in which the lines vary in intensity over the length according to local changes in the composition of the specimen along the path of the spark.

A diagram illustrating the basic electrical circuit for producing the discharge, and the mechanical means for controlling and guiding the spark across the sample, is shown in Fig. 1. The circuit is that of a relaxation oscillator. The high voltage direct current from a suitable source is passed through a control tube which regulates the rate of charging the capacitor. This control tube is operated under such conditions that the current passing through the tube is governed by the filament temperature alone. When the charge on the capacitor reaches the gap breakdown voltage, a spark bridges the gap and discharges the condenser. In order to prevent oscillation of the circuit and rapid polarity changes at the gap electrodes, Scheibe and Martin used a vacuum tube to shunt the current when the gap point is positive. Thus the spark should bridge the gap only when the specimen is positive and the guiding point negative. On half-waves of reversed polarity the current is shunted through the vacuum tube. The discharge is guided by a quartz tube with capillary opening, 0.01–0.03 mm diameter. The electrode inside this tube is a fine wire of aluminum, 0.010″ diameter, chosen because aluminum does not sputter appreciably under these conditions. The gap inside the guiding tube, from the end of the aluminum wire to the outside of the capillary tip, is made as short as possible, about 1 mm. The external gap from the quartz tip to the surface of the specimen is set in the range 0.1–0.4 mm. The specimen is clamped onto the rotating drum or the drum is machined from the specimen itself.

A diagram of a working circuit is shown in Fig. 2. The main discharge portion as shown in Fig. 1 is represented by heavy lines in this diagram and the supply circuits are shown in light lines. 60-cycle supply passes through a voltage regulator so that the entire apparatus is not subject to line voltage fluctuations. A conventional rectifier-filter circuit maintains the filamentary cathode of the 878 vacuum tube at −6 kv. This tube, the control tube, may be considered to be a high resistance of adjustable value. The filament is heated with d.c. The

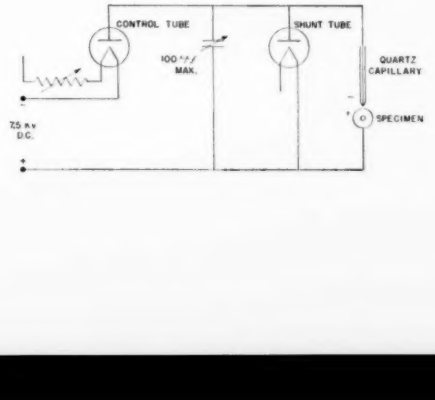

FIG. 1.

[1] G. Scheibe, Zeits. f. Angew. Chemie **50**, 288 (1937).
[2] G. Scheibe and J. Martin, Spectrochim. Acta **1**, 47 (1939).
[3] A. Henrici and G. Scheibe, "Chemische Spektralanalyse," *Physikalische Methoden der Analytischen Chemie*, Part III, p. 156.
[4] J. Martin, Thesis (Munich, 1937).

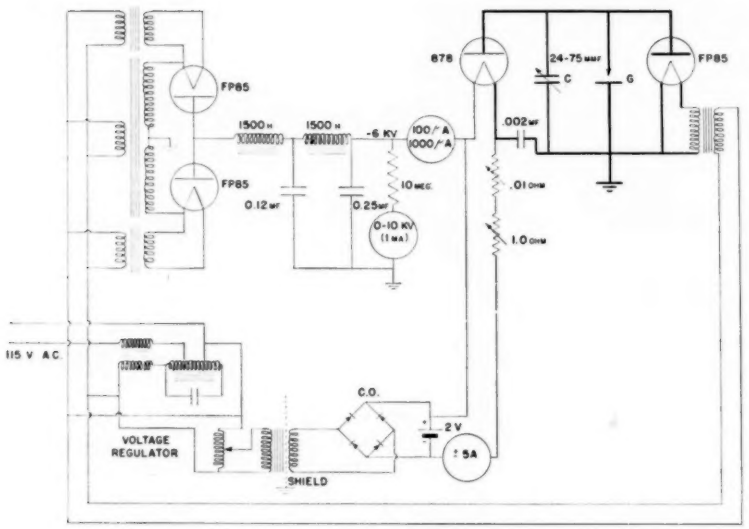

FIG. 2.

temperature of the filament is adjusted by means of the series resistance. To produce constancy of the discharge circuit and to avoid synchronization with the 120-cycle ripple frequency of the power sources, a storage battery is floated across the copper oxide rectifier. Effective filtration of the high voltage supply is also necessary.

Figure 3 illustrates the specimen holder used in our investigation. Provision was made for clamping sheet specimens onto the rotating drum by means of collars. Adjustment of the position of the sample is accomplished by rotation of the drum and movement of the drum and axle by the screw acting against the spring. Quartz guiding tubes were made by the method described in the thesis by Martin. A quartz tube is drawn down to a point, sealed off, and the tip is ground on fine emery paper until the desired bore in the taper is reached. The end of this quartz capillary is painted with a mixture of water glass and ferric oxide which is heat resistant and opaque. This coating screens off the light from the spark inside the tube. Adjustment of the gap length was made with the aid of a short focal length measuring telescope.

In the work by Scheibe and Martin, the spark served as its own slit. They removed the spectrograph slit and placed the spark in the plane of the slit, thus obtaining maximum light intensity from the weak source. We have found it more convenient to form an image of the spark its actual size at the slit. This method of illumination has proved satisfactory and does not cause much loss of intensity if the aperture of the condensing lens is the same as the aperture of the spectrograph. The use of the spectrograph slit also gives better definition of lines than the arrangement of Scheibe. The spectrograph is unaltered and is readily converted to use for ordinary spectroscopic work simply by changing the exterior lens system.

The complete assembly of our equipment is illustrated in Fig. 4. On the extreme left is the motor and gear system of an electric time-switch which moves the sample at a speed of 3 mm per hour. Next is shown the specimen holder with the built-in light used in making gap adjustments with the measuring telescope. The condensing lens is shown between the specimen holder and spectrograph slit. This lens is a quartz plano-convex condenser. The focal length used was corrected to 3000A. Therefore, the spark image at the slit was sharp only over a narrow range in the ultraviolet and the visible image on the slit was only a large blur. The spark length determined the length of line on the plate because no diaphragm was used to limit the slit length. In the wave-length range where the spark image at the slit was sharp, the lines on the plate were about 0.3 mm long for an exposure with stationary

plate. On the right behind the spectrograph the plate driving motor is located. This is a synchronous clock motor with gear system and is connected to the plate drive with chain and sprocket in such a manner that the plate is moved vertically at the rate of 18 mm per hour. The exposure time for a 0.3-mm line on the plate is, therefore, 1 minute for this rate of movement.

Samples of metal foil having a bright surface do not require any special preparation before they may be examined by this technique other than removing grease from the surface by washing with ether. However, most metal specimens will require polishing and we have followed the technique and recommended practices of preparing specimens for metallographic examination. We found it necessary to employ a procedure which would not remove inclusions from the metal while polishing. This preparation of the specimen is important and care must be taken that the feature to be studied is not destroyed by the polishing of the specimen—either by being torn out or by flow of the metal.

A synthetic sample prepared and examined in our laboratory is shown in Fig. 5. The specimen was a piece of polished sheet steel which had a streak of copper across the surface. This was prepared by wetting a thread with copper sulfate and placing it on the steel for a few seconds. A thin film of copper was plated onto the steel, the film being about 0.5 mm wide and so thin that it was transparent, but copper-colored. As seen in the photograph, the copper lines are strong where the spark crossed this narrow streak on the steel.

An interesting example of actual results obtained with this technique is illustrated by Figs. 6 and 7. For some time small crystalline inclusions had been observed in one of our steel samples. It was assumed that they were aluminum oxide because they were transparent crystals of high refractive index, high specific gravity, harder than the matrix and the steel was known to contain some aluminum oxide. In Fig. 6 an enlarged photograph of the track of the microspark on one of the specimens is shown with a particle marked by an arrow. The actual distance on the specimen of the two black lines parallel with the track is 0.2 mm. Magnesium lines were found in the spectrum where the spark struck this particle. A photograph of the spectrogram is shown in Fig. 7 and these magnesium lines are very evident.

These few examples serve to illustrate the type of results obtained with this equipment. The apparatus does work and has proved its value, but there are many interesting features and problems connected with the electrical circuit which need investigation. One of the first problems which we encountered was the peculiar type of spectrum given by the discharge. It is not an arc spectrum or a condensed spark spectrum although it contains a few lines from both. The iron spectrum given by this discharge appears to consist of lines originating in high energy changes. The few arc lines which appear are very weak. The iron spectrum from the microspark is shown in Fig. 8 in between iron spectra from a d.c. arc and a condensed spark. In the microspark spectrum, most of the iron lines lie in the range 2400A–3100A. This region contains most of the intense iron spark lines, and we have concluded that the discharge yields an almost pure spark spectrum. In the examination of an etched aluminum foil which had boron compounds on the surface, we obtained the boron doublet 2067.88A and 2066.41A as intense lines. These lines are supposed to originate in the higher state B III. The spectrographic

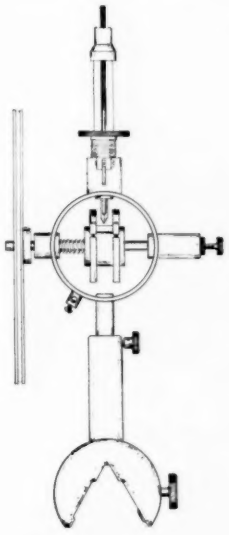

FIG. 3.

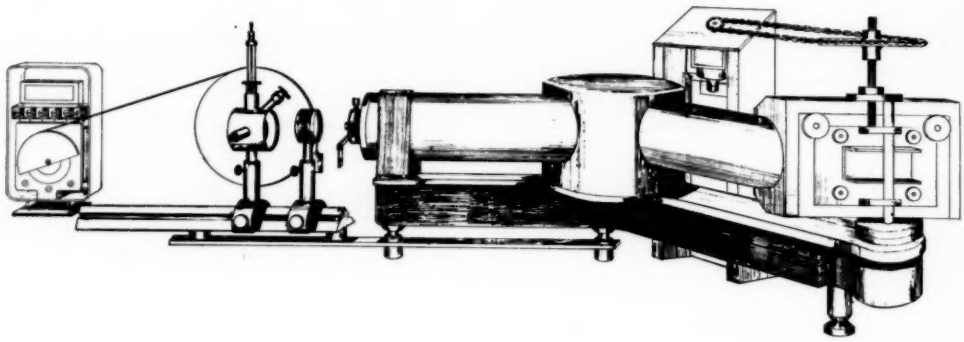

FIG. 4.

plate used was an Eastman Type I-O, *not* sensitized for the ultraviolet. On the same plate, aluminum 1989.8A (Al II) appeared with equal intensity. The appearance of these lines on an unsensitized plate is further evidence that the spark produces an intense spectrum corresponding to high energy states. Using the same plates, spectrograph, etc., with the usual excitation of the d.c. arc or Feussner condensed spark, we do not obtain lines below 2200A (lines as low as 1980A can be obtained using a sensitized plate). Martin has published some data in his thesis on the relative intensity of lines in an arc spectrum and in a spectrum from this microspark. Martin discussed the spectra of copper and aluminum and although there are some intensity changes between arc and microspark spectra for these elements, the iron spectrum is changed much more radically than these simpler spectra.

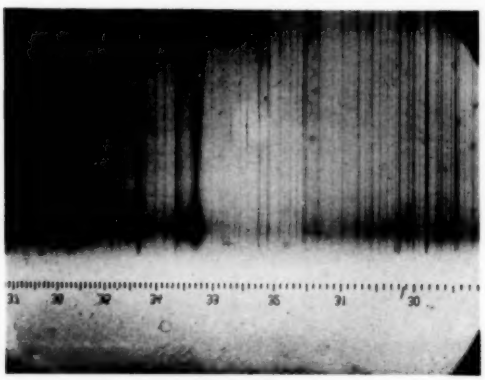

FIG. 5.

As a result of this different type of spectrum yielded by the discharge, it was soon found that the discharge is not as satisfactory for detecting traces of impurities in iron or steel as is the ordinary d.c. arc or even the condensed spark. In an effort to increase the sensitivity of the discharge and to change the type of spectrum obtained from it if possible, we have made a rather extensive study of the characteristics of the spark and of the effect of variations in the circuit on the spectrum produced.

In Table I the most important electrical characteristics of the spark are tabulated.

TABLE I. *Characteristics of the electrical circuit.*

Potential drop across gap	2000 volts
Peak current per discharge	15 amp.
Oscillation frequency of each discharge	20 Mc
Time to reach peak current	0.01 microsec.
Frequency of discharges	0–10,000/sec.

The potential drop of 2000 v, or more, across the gap is much higher than that of a d.c. arc. This potential drop, which exists at the instant of breakdown, gives rise to high velocity electrons. The resistance of the gap, and potential drop across the gap, drops to almost zero during the spark. The current flowing in the circuit is then largely determined by the surge impedance of the circuit. It is difficult to make the inductance in the circuit, that due to the length of leads, less than 1 microhenry.

The physical nature of the spark is determined by these circuit characteristics. The high voltage drop across the gap is the only explanation we have found for the enhanced spectra obtained from this discharge. In the usual d.c. arc, the potential drop between the electrodes is 125

volts or less. Kaiser and Walraff [5] have shown that in the case of a condensed spark, there is a high potential ignition discharge lasting about 0.01 microsecond, after which the potential drop across the gap falls to about 50 volts and remains at this low value during the oscillating discharge. It is evident then that this microspark discharge corresponds to the individual high potential ignition sparks in a condensed spark without the subsequent burning period under arc conditions. There is no time when the discharge is burning steadily at the low voltage which is necessary for arc spectra. We believe, therefore, that the enhanced spectra obtained from the microspark originate in the short high potential discharges. The lack of sensitivity of this microspark for small amounts of impurities in a metal is probably explained by these discharge characteristics. First, the discharge produces enhanced spectra and only a few elements have strong spark lines falling in the ultraviolet region 2100A–4000A where we are working. If the discharge could be used in conjunction with a vacuum spectrograph so that the shorter wave-lengths were accessible, more sensitive spark lines might be obtained. Secondly, the discharge consumes only a minute quantity of electrode material. This means that if it strikes a region of high local concentration of impurity, the spectrum will be recorded; but if the impurity is in solid solution and the concentration is low, it may or may not be recorded, depending upon the sensitivity of its spectrum lines with this discharge.

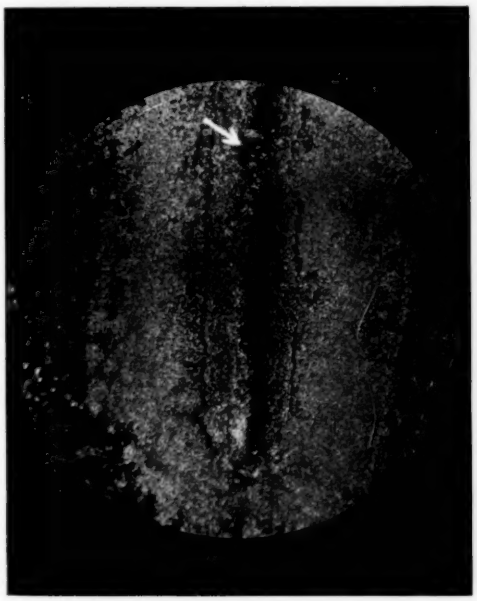

FIG. 6.

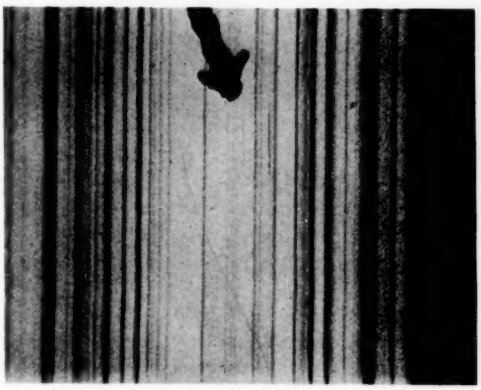

FIG. 7.

At this point it is probably evident that there are apparently two conflicting but desirable features to be sought in the microspark source for local analysis: (1) high sensitivity for traces of impurities; (2) high degree of localization and very little erosion of the specimen. The electrical circuit which Scheibe has developed seems to attain a median between these two extremes. If the source has to be very sensitive for traces, the usual technique is to use a very hot, low voltage arc which melts the specimen. If one desires localization, he usually seeks a high voltage spark. This microspark circuit seems to yield a highly localized source with fair sensitivity, and an attempt to increase one feature will usually mean a decrease in the other. We have found that in the steels with which we have been working we have been able to detect concentrations of aluminum above 0.3 percent and manganese above 0.05 percent, but we have not been able to detect nickel or chromium at 0.3 percent, nor copper at 0.2 percent.

The effects of varying the circuit constants and the atmosphere, pressure, gap length, etc., have

[5] Kaiser and Walraff, Ann. d. Physik **34**, 297 (1939).

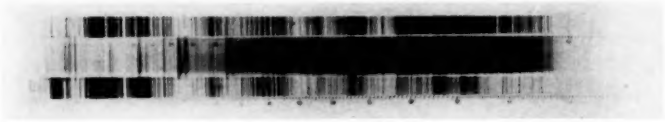

FIG. 8.

been investigated. Scheibe and Martin used nitrogen and argon atmospheres and worked at lowered pressures. Our experience has been that pure nitrogen causes more intense bands to appear in the spectrum than air. We have tried helium and found that the discharge changed to a helium glow and gave very few metal lines in the spectrum. Apparently the main object in the use of the inert atmosphere by Scheibe was to prevent oxidation of metals such as tin, magnesium, aluminum, etc., which he was studying. We have found that iron and aluminum specimens are oxidized where the spark strikes when run in air, but the oxidation does not interfere seriously. We have also tried lowering the pressure inside the quartz capillary by pumping through a side-arm sealed on the glass insulating section above the capillary, outside the capillary by closing the vessel and pumping, and both at once. The effect each time was to cause the spark to change to a glow discharge and give mainly a gas spectrum with very few metal lines. Therefore, our conclusions have been that it is much simpler to operate the spark in air at atmospheric pressure and the results are more satisfactory, when one is interested in high sensitivity for traces.

From the standpoint of the electrical circuit, increasing the capacity from 25$\mu\mu$f to 100$\mu\mu$f with the same steady state current supplying the capacitor causes an increase in intensity of the spectrum as a whole. With a given capacitance, increasing the steady state supply current from 1 to 100 microamperes causes an increase of spectrum intensity. No change in character of the spectrum is brought about by these changes in the electrical circuit.

Scheibe used a grounded conducting ring of wire around the glass envelope of the control tube to drain off creepage currents which made the spark unsteady. We have found this unnecessary if the glass is kept clean.

The shunt tube is intended to carry the half-wave of current of undesired polarity. Since it is of the high vacuum type, its time lag is sufficiently low. Its steady state rating is 0.1 ampere. However, due to the storage factor of the space charge in the immediate vicinity of the cathode, its current carrying capacity for a transient of so short a duration is many times greater. The tube will damp the oscillations of each discharge even if it does not stop them entirely. We have found from direct measurement that there is somewhat less average current passing through the gap when the tube is in the circuit than when it is out. However, we have not found any evidence of change in the character of the spectrum with or without this tube in the circuit. It has not been thought necessary to attempt to employ a tube of greatly increased current carrying capacity and obtain critical damping since spectral results were not altered by critically damping the oscillatory circuit with external resistance.

CONCLUSIONS

Our investigation has demonstrated that the microspark can be advantageously used in the manner described by Scheibe to investigate metallic surfaces. It is potentially a very powerful tool for the study of metallographic problems since it permits the actual analysis of phases as they are seen under the microscope or the separation of phases chemically different which cannot be distinguished by appearance.

The chief limitation of the microspark at the present time is in the detection of low *concentrations* of materials, but it seems certain that improvements can be made in this respect by methods which we now have under investigation.

Zeeman Effect Data and Further Classification of the First Spark Spectrum of Cerium—Ce II

GEORGE R. HARRISON, WALTER E. ALBERTSON AND NORMAN F. HOSFORD
George Eastman Research Laboratories of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts

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The light emitted by cerium atoms in magnetic fields up to 96,400 oersteds has been photographed at high spectrographic resolution over the range 2500 to 7000A. Interpretable Zeeman patterns of 427 Ce II lines have been reduced, and from them g and J values have been determined for 280 levels. These data have been used to check and extend the classification of Ce II, in which 3600 lines are now assigned to transitions between pairs of 316 levels. The energy system of Ce II consists of two groups of levels which have not yet been connected. Group I is believed to be the lower of the two by approximately 5000 wave numbers. This group consists of levels arising from the electron configurations $4f^26s$, $4f^25d$, $4f^26p$ and $4f^3$. Complete assignments of quantum numbers have been given to all levels in this group on the basis of Zeeman effect studies, combinations, intervals and intensities. All terms have been assigned to electron configurations and parent terms. Group II consists of energy levels believed to originate from the electron configurations $4f5d6s$, $4f5d^2$, $4f5d6p$, $4f6s6p$, and others. g values to 3 figures after the decimal are well established for many of these levels, but only a few levels are given L and S assignments, which are tentative. Interactions among these levels are strong, and assignment of individual quantum numbers will probably have little significance. Data are now available for complete tests of the g sum rule in a number of cases. It is found to be exact in cases where all perturbing terms are known. g values obtained from different lines for the same energy level are consistent to within 0.003 unit on the average.

IN a preliminary report on the classification of the first spark spectrum of cerium (Ce II)[1] 584 lines were accounted for as transitions between 31 lower and 51 upper states. Further progress has now been made in the analysis of this extremely complex spectrum, more than 3600 lines having been assigned to transitions between pairs of 316 levels.

The analysis has been advanced by the further use of the mechanical interval sorter and interval recorder,[2] and has been greatly expedited by the availability of new Zeeman effect measurements which we have made. Results obtained from these measurements will be discussed in the present paper. Detailed consideration of all of the term assignments will be reserved for the complete publication of the Ce II analysis.

ZEEMAN EFFECT MEASUREMENTS

The Zeeman effect of cerium has been studied by Van de Vliet,[3] who reported measurements on some fifty-four Ce II lines (other than unresolved triplets) and verified our statement[1] that the purported term array for Ce II published by Haspas[4] is fortuitous and without meaning. Van de Vliet used fields up to 43,000 oersteds.

The availability of a Bitter magnet in this laboratory, giving fields up to 100,000 oersteds,[5] has now made possible the complete magnetic resolution of several hundred lines produced by the cerium arc in air, and the partial resolution of thousands.

For burning in the horizontal arc, one part of cerium chloride was mixed with four parts of silver powder and compressed at pressures of 20,000 lb. per sq. inch to form a rod $\frac{1}{8}''$ in diameter, which when sintered was strong enough to be turned in a lathe. The arc itself has been described previously.[5]

Exposures were made simultaneously on the three concave gratings previously described.[6] The spectrum from 2085 to 8000A was covered at dispersions ranging from 0.4 to 1.6A/mm, but very few Ce II lines were found at wave-lengths shorter than 2500A or longer than 5000A. The p and n components of polarization of the transverse Zeeman effect were photographed separately but in juxtaposition on the same plates, so

[1] W. E. Albertson and G. R. Harrison, Phys. Rev. **52**, 1209 (1937).
[2] G. R. Harrison, Rev. Sci. Inst. **3**, 753 (1932); **4**, 581 (1933); J. Opt. Soc. Am. **28**, 290 (1938).
[3] H. J. Van de Vliet, "Het Zeeman Effect van de Spectraallijnen van Cerium en Neodymium," Thesis, Amsterdam, 1939.
[4] K. Haspas, Zeits. f. Physik **96**, 410 (1935).
[5] G. R. Harrison and F. Bitter, Phys. Rev. **57**, 15 (1940).
[6] G. R. Harrison and J. R. McNally, Jr., Phys. Rev. **58**, 703 (1940).

TABLE I. *Zeeman patterns of* Ce II *lines.*

Wave-Length	Int.	Combination	Type	Sep.	p	n	g_1	g_2
6986.07	15	$c^4H_{6\frac{1}{2}}-z^4H^\circ_{6\frac{1}{2}}$	6	57	(372)	1178	1.206	1.150
6760.81	4	$c^4H_{5\frac{1}{2}}-z^4I^\circ_{6\frac{1}{2}}$	7b	—	(0)	1185	1.120	(1.130)
6744.70	6	$c^4H_{4\frac{1}{2}}-z^4G^\circ_{3\frac{1}{2}}$	5	66	(32)	1182	.961	.895
6466.898	5	$c^4H_{3\frac{1}{2}}-z^4G^\circ_{4\frac{1}{2}}$	7b	—	(0)	1198	1.128	(1.112)
6433.41	2	$73^\circ_{4\frac{1}{2}}-114_{4\frac{1}{2}}$	7b	—	(0)	921	.921	.921
6422.937	2	$c^4H_{6\frac{1}{2}}-z^4I^\circ_{7\frac{1}{2}}$	7b	—	(0)	1205	1.205	1.205
6371.105	10	$b^4F_{1\frac{1}{2}}-z^4G^\circ_{2\frac{1}{2}}$	5	120	(61)	773	.466	.586
6299.514	4	$c^4H_{6\frac{1}{2}}-z^4G^\circ_{5\frac{1}{2}}$	6	—	(93)	1134	1.196	(1.207)
6272.051	20	$c^4H_{3\frac{1}{2}}-z^4G^\circ_{2\frac{1}{2}}$	5	105	(—)	959	.699	.594
6232.452	8	$b^4H_{3\frac{1}{2}}-z^4I^\circ_{4\frac{1}{2}}$	5	108	(54)	1159	.693	.801
6115.15	—	$71^\circ_{3\frac{1}{2}}-116_{3\frac{1}{2}}$	6	126	(435)	998	1.036	.910
6098.336	10	$c^4H_{5\frac{1}{2}}-y^2G^\circ_{4\frac{1}{2}}$	7b	—	(0)	1210	1.122	(1.102)
6043.386	30	$b^4H_{3\frac{1}{2}}-z^4H^\circ_{3\frac{1}{2}}$	6	—	—	697	(710)	.684
5995.367	3	$b^4H_{4\frac{1}{2}}-z^4I^\circ_{5\frac{1}{2}}$	5	—	(118)	1093	.987	(1.006)
5975.830	15	$b^4H_{4\frac{1}{2}}-z^4H^\circ_{4\frac{1}{2}}$	6	—	(108)	957	.969	.945
5783.988	3	$b^4H_{5\frac{1}{2}}-z^4I^\circ_{6\frac{1}{2}}$	7b	—	(0)	1165	1.124	(1.130)
5715.286	3	$c^4H_{5\frac{1}{2}}-y^4G^\circ_{5\frac{1}{2}}$	5	58	(29)	878	.672	.614
5711.447	8	$b^4H_{3\frac{1}{2}}-z^2H^\circ_{4\frac{1}{2}}$	6	—	(76)	1116	1.122	1.108
5668.901	20	$a^4I_{4\frac{1}{2}}-z^4I^\circ_{5\frac{1}{2}}$	6	—	(107)	774	.763	.787
5613.698	5	$a^2I_{6\frac{1}{2}}-z^2H^\circ_{5\frac{1}{2}}$	7b	—	(0)	1081	1.101	(1.105)
5561.47	—	$b^4H_{3\frac{1}{2}}-z^4G^\circ_{4\frac{1}{2}}$	5	—	—	1178	(1.127)	1.116
5518.491	12	$a^4G_{3\frac{1}{2}}-z^4H^\circ_{4\frac{1}{2}}$	7b	—	(0)	835	.971	(.941)
5516.081	6	$b^4H_{4\frac{1}{2}}-z^4G^\circ_{5\frac{1}{2}}$	7b	—	(0)	1159	1.200	(1.207)
5409.224	50	$a^4I_{4\frac{1}{2}}-z^4I^\circ_{5\frac{1}{2}}$	6	—	(174)	994	.978	1.010
5393.391	30	$a^4I_{4\frac{1}{2}}-z^4H^\circ_{4\frac{1}{2}}$	7b	—	(0)	1009	.953	(.941)
5353.534	50	$a^4K_{4\frac{1}{2}}-z^4I^\circ_{4\frac{1}{2}}$	7b	—	(0)	831	.803	(.797)
5347.806	8	$42^\circ_{3\frac{1}{2}}-101_{4\frac{1}{2}}$	5	—	(0)	1038	.954	(.935)
5330.582	25	$b^2H_{4\frac{1}{2}}-z^4I^\circ_{4\frac{1}{2}}$	6	70	(384)	852	.888	.817
5274.244	50	$a^4K_{4\frac{1}{2}}-z^4I^\circ_{5\frac{1}{2}}$	4	—	(—)	831	.979	(1.006)
5265.710	15	$a^4I_{6\frac{1}{2}}-z^4I^\circ_{4\frac{1}{2}}$	6	—	(161)	1116	1.106	1.130
5205.521	8	$a^4I_{6\frac{1}{2}}-z^2H^\circ_{5\frac{1}{2}}$	7b	—	(0)	1104	1.105	(1.105)
5191.675	30	$b^2H_{4\frac{1}{2}}-z^4H^\circ_{3\frac{1}{2}}$	5	172	(93)	1485	.887	.715
5117.175	20	$a^4I_{3\frac{1}{2}}-z^4I^\circ_{3\frac{1}{2}}$	7b	—	(0)	1200	1.195	(1.205)
5076.474	10	$a^4G_{5\frac{1}{2}}-z^4G^\circ_{5\frac{1}{2}}$	6	—	(99)	1227	1.235	1.217
5044.008	25	$a^4K_{3\frac{1}{2}}-z^4H^\circ_{4\frac{1}{2}}$	4	65	(32)	667	1.090	1.155
5011.765	12	$a^4G_{2\frac{1}{2}}-z^4G^\circ_{3\frac{1}{2}}$	6	—	(100)	613	.633	.593
4773.494	—	$a^4F_{4\frac{1}{2}}-z^4G^\circ_{2\frac{1}{2}}$	5	174	(90)	852	.416	.590
4714.83	—	$a^4F_{3\frac{1}{2}}-z^4G^\circ_{4\frac{1}{2}}$	4	102	—	—	(1.213)	1.111
4713.996	10	$62^\circ_{5\frac{1}{2}}-137_{5\frac{1}{2}}$	6	—	(269)	1133	1.108	1.158
4684.605	8	$52^\circ_{6\frac{1}{2}}-120_{5\frac{1}{2}}$	5	—	(0)	1149	1.089	(1.079)
4654.286	6	$20^\circ_{1\frac{1}{2}}-103_{1\frac{1}{2}}$	6	—	(245)	609	.529	.691
4628.160	20	$a^4H_{4\frac{1}{2}}-z^4I^\circ_{4\frac{1}{2}}$	6	154	(691)	871	.949	.795
4624.899	8	$a^2F_{2\frac{1}{2}}-y^2G^\circ_{4\frac{1}{2}}$	5	—	(0)	1327	(1.026)	1.093
4606.402	12	$a^2H_{5\frac{1}{2}}-z^4I^\circ_{6\frac{1}{2}}$	5	—	(0)	1323	1.095	(1.130)
4593.932	30	$a^4H_{4\frac{1}{2}}-z^4I^\circ_{5\frac{1}{2}}$	7b	—	(0)	1277	(.932)	.995
4582.502	10	$a^2H_{4\frac{1}{2}}-z^4H^\circ_{4\frac{1}{2}}$	7b	—	(0)	939	.939	.939
4572.277	35	$a^4H_{5\frac{1}{2}}-z^4I^\circ_{5\frac{1}{2}}$	6	119	607	1069	1.124	1.905
4565.842	12	$a^4F_{4\frac{1}{2}}-y^4G^\circ_{4\frac{1}{2}}$	6	127	(—)	1178	1.247	1.120
4562.360	40	$a^4H_{3\frac{1}{2}}-z^4I^\circ_{4\frac{1}{2}}$	5	122	(61)	1227	.678	.800
4560.280	25	$a^2H_{5\frac{1}{2}}-z^2H^\circ_{5\frac{1}{2}}$	6	—	(234)	1106	1.095	1.116
4551.297	20	$42^\circ_{5\frac{1}{2}}-115_{4\frac{1}{2}}$	—	—	—	934	.956	(.968)
4549.644	6	$a^2G_{3\frac{1}{2}}-z^2F^\circ_{3\frac{1}{2}}$	6	—	(217)	1047	1.014	.1078
4539.072	5	$a^2G_{4\frac{1}{2}}-z^4F^\circ_{4\frac{1}{2}}$	5	79	—	—	(1.014)	1.093
4528.472	30	$a^4H_{5\frac{1}{2}}-z^4I^\circ_{6\frac{1}{2}}$	6	90	(567)	—	(1.223)	1.133
4527.348	50	$8^\circ_{4\frac{1}{2}}-101_{4\frac{1}{2}}$	6	—	(389)	976	1.019	.933
4523.077	35	$a^4H_{4\frac{1}{2}}-z^4H^\circ_{3\frac{1}{2}}$	5	238	(117)	1779	.948	.710
4519.592	8	$a^2G_{4\frac{1}{2}}-z^2F^\circ_{3\frac{1}{2}}$	5	106	(54)	1565	1.202	1.096
4508.083	8	$30^\circ_{2\frac{1}{2}}-107_{3\frac{1}{2}}$	4	—	(0)	991	1.022	(1.013)
4497.849	30	$a^4F_{2\frac{1}{2}}-y^4G^\circ_{2\frac{1}{2}}$	6	373	(930)	800	.986	.613
4495.380	4	$30^\circ_{2\frac{1}{2}}-108_{2\frac{1}{2}}$	7b	—	(0)	1051	1.051	1.051
4486.909	40	$6^\circ_{4\frac{1}{2}}-101_{4\frac{1}{2}}$	6	106	(482)	986	1.040	.934
4483.900	20	$a^4H_{6\frac{1}{2}}-z^2H^\circ_{5\frac{1}{2}}$	5	122	(62)	1898	1.230	1.108
4479.432	4	$13^\circ_{2\frac{1}{2}}-103_{1\frac{1}{2}}$	5	208	(152)	1444	.997	.699
4479.359	40	$26^\circ_{4\frac{1}{2}}-105_{4\frac{1}{2}}$	6	—	(210)	965	.988	.942
4471.240	35	$a^2H_{4\frac{1}{2}}-z^2H^\circ_{4\frac{1}{2}}$	7b	—	(0)	931	(.932)	.930
4467.537	5	$a^4F_{4\frac{1}{2}}-z^4G^\circ_{5\frac{1}{2}}$	4	—	(0)	1075	1.249	(1.217)
4463.410	35	$a^4F_{2\frac{1}{2}}-z^4D^\circ_{1\frac{1}{2}}$	5	308	(153)	1450	.987	.679
4461.138	50	$a^2H_{5\frac{1}{2}}-z^2I^\circ_{5\frac{1}{2}}$	6	67	—	—	(1.088)	1.021
4460.213	60	$a^4H_{3\frac{1}{2}}-z^4H^\circ_{3\frac{1}{2}}$	6	—	(117)	691	.674	.708
4450.732	35	$a^4H_{3\frac{1}{2}}-z^2H^\circ_{4\frac{1}{2}}$	5	193	(97)	1994	1.126	.933
4449.336	50	$29^\circ_{3\frac{1}{2}}-109_{3\frac{1}{2}}$	6	—	(203)	1082	1.104	1.059
4444.393	30	$a^4F_{4\frac{1}{2}}-y^4G^\circ_{2\frac{1}{2}}$	5	200	(103)	906	.406	.606
4443.743	18	$33^\circ_{3\frac{1}{2}}-115_{4\frac{1}{2}}$	4	—	(0)	914	(1.001)	982
4440.883	20	$44^\circ_{2\frac{1}{2}}-124_{1\frac{1}{2}}$	7b	—	(0)	973	.978	(.981)
4428.438	18	$22^\circ_{3\frac{1}{2}}-105_{4\frac{1}{2}}$	4	—	(0)	931	.966	(.958)
4423.678	25	$a^4F_{3\frac{1}{2}}-y^2G^\circ_{3\frac{1}{2}}$	6	231	(804)	1088	1.203	.972
4418.784	40	$a^4H_{6\frac{1}{2}}-z^4H^\circ_{6\frac{1}{2}}$	6	—	(384)	1186	1.218	1.153
4416.902	25	$22^\circ_{3\frac{1}{2}}-106_{3\frac{1}{2}}$	6	122	(429)	—	(.966)	.844
4407.278	40	$35^\circ_{3\frac{1}{2}}-117_{4\frac{1}{2}}$	5	200	(99)	1092	1.088	.888
4399.203	35	$10^\circ_{2\frac{1}{2}}-102_{2\frac{1}{2}}$	6	201	(494)	861	.960	.760
4398.787	20	$24^\circ_{3\frac{1}{2}}-107_{3\frac{1}{2}}$	6	118	(420)	1078	1.136	1.018
4394.779	30	$65^\circ_{2\frac{1}{2}}-171_{3\frac{1}{2}}$	5	62	(31)	1167	.950	1.012
4391.661	40	$9^\circ_{1\frac{1}{2}}-102_{2\frac{1}{2}}$	5	242	(120)	1127	.521	.763
4390.279	10	$70^\circ_{3\frac{1}{2}}-194_{4\frac{1}{2}}$	7b	—	(0)	1078	1.078	1.078
4388.007	8	$a^4H_{6\frac{1}{2}}-z^2I^\circ_{5\frac{1}{2}}$	5	207	(108)	2364	1.227	1.020
4386.835	15	$4^\circ_{3\frac{1}{2}}-101_{4\frac{1}{2}}$	5	119	(—)	—	(.806)	.925
4382.167	40	$a^4H_{5\frac{1}{2}}-z^4H^\circ_{5\frac{1}{2}}$	6	84	(466)	1078	1.125	1.041
4380.060	30	$30^\circ_{2\frac{1}{2}}-113_{3\frac{1}{2}}$	5	197	(101)	—	(1.037)	.840
4375.918	40	$19^\circ_{2\frac{1}{2}}-105_{4\frac{1}{2}}$	5	74	(37)	1212	.879	.953
4373.818	40	$26^\circ_{4\frac{1}{2}}-109_{4\frac{1}{2}}$	5	71	(33)	—	(.978)	1.049
4372.401	35	$23^\circ_{2\frac{1}{2}}-107_{3\frac{1}{2}}$	5	253	(125)	1650	.765	1.018
4368.234	8	$52^\circ_{6\frac{1}{2}}-130_{6\frac{1}{2}}$	6	—	(127)	1113	1.103	1.123
4364.658	30	$19^\circ_{3\frac{1}{2}}-106_{3\frac{1}{2}}$	6	—	(116)	853	.869	.837
4361.661	18	$22^\circ_{3\frac{1}{2}}-107_{4\frac{1}{2}}$	6	—	(170)	973	.949	.999
4360.16	25	$45^\circ_{1\frac{1}{2}}-129_{1\frac{1}{2}}$	6	204	(309)	806	.910	.706
4349.789	40	$35^\circ_{3\frac{1}{2}}-120_{4\frac{1}{2}}$	6	—	(68)	1083	1.089	1.077
4345.963	6	$62^\circ_{5\frac{1}{2}}-150_{4\frac{1}{2}}$	5	80	40	—	(1.068)	1.148
4339.317	25	$a^2G_{4\frac{1}{2}}-z^2G^\circ_{4\frac{1}{2}}$	6	—	(294)	—	(1.193)	1.128
4337.777	25	$10^\circ_{2\frac{1}{2}}-103_{4\frac{1}{2}}$	5	266	(133)	1358	.954	.688
4336.255	25	$36^\circ_{4\frac{1}{2}}-123_{3\frac{1}{2}}$	5	116	(58)	1646	1.240	1.124
4330.445	50	$9^\circ_{1\frac{1}{2}}-103_{1\frac{1}{2}}$	6	175	(263)	602	.516	.691
4326.826	15	$39^\circ_{4\frac{1}{2}}-124_{4\frac{1}{2}}$	6	260	390	1105	1.238	.978
4320.523	50	$16^\circ_{2\frac{1}{2}}-105_{4\frac{1}{2}}$	4	132	(65)	496	1.088	.956
4310.699	30	$19^\circ_{3\frac{1}{2}}-107_{3\frac{1}{2}}$	6	107	—	—	.910	1.017
4309.739	25	$16^\circ_{3\frac{1}{2}}-106_{3\frac{1}{2}}$	6	244	(848)	963	1.084	.840
4300.331	40	$15^\circ_{4\frac{1}{2}}-105_{4\frac{1}{2}}$	6	—	(130)	964	.979	.949
4299.092	5	$19^\circ_{3\frac{1}{2}}-108_{2\frac{1}{2}}$	4	184	—	427	.887	1.071
4296.786	5	$47^\circ_{6\frac{1}{2}}-139_{6\frac{1}{2}}$	6	100	(—)	1153	1.214	1.114
4296.680	40	$a^4H_{4\frac{1}{2}}-z^4H^\circ_{4\frac{1}{2}}$	7b	—	(0)	947	.947	.947
4289.938	50	$11^\circ_{2\frac{1}{2}}-104_{3\frac{1}{2}}$	6	103	(363)	878	.834	.937
4289.454	25	$15^\circ_{4\frac{1}{2}}-106_{3\frac{1}{2}}$	6	142	(497)	1072	.995	.843
4285.366	25	$a^2H_{5\frac{1}{2}}-y^2G^\circ_{4\frac{1}{2}}$	4	—	(0)	1082	(1.088)	1.089
4280.141	15	$39^\circ_{4\frac{1}{2}}-126_{2\frac{1}{2}}$	4	208	(107)	749	1.259	1.051
4278.866	20	$8^\circ_{4\frac{1}{2}}-104_{2\frac{1}{2}}$	5	82	(39)	1313	1.032	.950
4270.189	25	$26^\circ_{4\frac{1}{2}}-115_{4\frac{1}{2}}$	7b	—	(0)	976	.976	.976
4264.370	10	$51^\circ_{3\frac{1}{2}}-144_{4\frac{1}{2}}$	4	72	(36)	—	(1.159)	1.087
4263.427	40	$50^\circ_{3\frac{1}{2}}-158_{2\frac{1}{2}}$	5	108	37	1606	1.336	1.224
4261.164	18	$a^4F_{4\frac{1}{2}}-y^4G^\circ_{3\frac{1}{2}}$	5	135	(68)	1734	1.262	1.127
4259.748	15	$50^\circ_{3\frac{1}{2}}-144_{4\frac{1}{2}}$	5	192	(100)	—	(1.266)	1.074
4257.121	20	$16^\circ_{3\frac{1}{2}}-107_{3\frac{1}{2}}$	6	—	(240)	1005	1.099	1.011
4255.784	40	$36^\circ_{4\frac{1}{2}}-125_{4\frac{1}{2}}$	6	115	(519)	1632	1.241	1.126
4253.356	40	$17^\circ_{1\frac{1}{2}}-108_{2\frac{1}{2}}$	4	165	(83)	817	1.226	1.061
4246.711	30	$5^\circ_{1\frac{1}{2}}-103_{1\frac{1}{2}}$	4	298	(149)	541	.988	.690
4242.723	15	$6^\circ_{4\frac{1}{2}}-104_{3\frac{1}{2}}$	5	103	(53)	1407	1.050	947
4239.912	35	$a^4H_{3\frac{1}{2}}-z^4H^\circ_{4\frac{1}{2}}$	5	274	(138)	1904	.667	.941
4237.205	8	$69^\circ_{3\frac{1}{2}}-200_{2\frac{1}{2}}$	4	—	(0)	1122	1.128	(1.131)
4236.023	30	$27^\circ_{2\frac{1}{2}}-118_{2\frac{1}{2}}$	6	108	267	964	1.020	.913
4234.727	12	$46^\circ_{4\frac{1}{2}}-140_{4\frac{1}{2}}$	7b	—	(0)	1228	1.228	1.228
4227.746	40	$a^2H_{4\frac{1}{2}}-z^2H^\circ_{5\frac{1}{2}}$	5	174	(89)	1889	.931	1 105
4222.599	80	$2^\circ_{4\frac{1}{2}}-101_{4\frac{1}{2}}$	6	—	(65)	941	.948	.934
4217.591	25	$a^4F_{3\frac{1}{2}}-y^4G^\circ_{3\frac{1}{2}}$	6	103	(372)	1160	1.216	1.112
4213.036	15	$33^\circ_{3\frac{1}{2}}-125_{4\frac{1}{2}}$	5	128	(67)	1576	1.000	1.128
4209.409	25	$a^4H_{3\frac{1}{2}}-z^2H^\circ_{4\frac{1}{2}}$	6	—	(79)	1114	1.121	1.107
4202.944	40	$25^\circ_{2\frac{1}{2}}-116_{3\frac{1}{2}}$	5	169	(84)	—	(.733)	.902
4198.724	8	$a^4H_{4\frac{1}{2}}-z^2H^\circ_{4\frac{1}{2}}$	6	116	(53)	935	.941	.929
4198.669	3	$a^2H_{5\frac{1}{2}}-z^4H^\circ_{5\frac{1}{2}}$	6	—	(607)	—	1.086	1.202

TABLE I.—*Continued.*

Wave-Length	Int.	Combination	Type	Sep.	p	n	g_1	g_2
4193.874	35	$24°_{3}-116_{2}$	6	235	(811)	1022	1.138	.902
4187.323	35	$24°_{3}-117_{4}$	5	242	(114)	—	1.140	.898
4176.080	12	$19°_{3}-115_{4}$	5	89	(43)	1289	.887	.976
4172.161	18	$12°_{3}-104_{4}$	5	184	(88)	—	(1.123)	.939
4169.773	12	$38°_{4}-131_{3}$	5	139	(73)	—	1.191	1.051
4167.804	12	$a^{3}F_{3}-z^{4}F°_{4}$	5	66	—	1259	1.026	1.092
4153.516	20	$a^{2}G_{4}-y^{2}H°_{4}$	5	128	(65)	1601	1.023	1.151
4151.175	18	$a^{4}F_{2}-z^{4}F°_{1}$	5	123	(63)	1175	.991	.868
4145.532	6	$a^{2}F_{3}-z^{4}F°_{3}$	6	—	(182)	1052	1.026	1.078
4138.901	25	$a^{2}F_{3}-z^{4}D°_{2}$	6	204	(503)	996	.896	1.101
4136.234	25	$26°_{4}-120_{3}$	5	98	(49)	1522	.983	1.081
4134.995	10	$a^{2}H_{4}-z^{4}G°_{4}$	6	170	(766)	1001	.924	1.094
4134.492	10	$a^{4}H_{3}-z^{2}H°_{4}$	5	252	(—)	—	(.669)	.921
4132.398	30	$a^{2}H^{1}_{5}-z^{2}I°_{5}$	5	88	(—)	1420	.933	1.021
4129.426	8	$66°_{3}-193_{2}$	5	198	(99)	—	(1.195)	.997
4125.443	20	$26°_{3}-121_{2}$	6	94	(232)	772	.725	.819
4120.706	25	$26°_{4}-122_{4}$	6	—	(85)	985	.976	.994
4129.176	5	$42°_{3}-139_{4}$	5	166	(79)		(.955)	1.121
4123.872	25	$47°_{4}-151_{6}$	6	—	(68)	1214	1.219	1.207
4120.829	25	$8°_{4}-105_{4}$	6	63	(283)	995	1.024	.961
4119.877	20	$10°_{3}-106_{3}$	4	110	(54)	—	(.957)	.847
4119.784	20	$a^{4}F_{4}-z^{4}F°_{4}$	6	149	(645)	1171	1.249	1.110
4119.015	25	$24°_{3}-123_{3}$	7b	—	(0)	1133	1.133	1.133
4117.288	20	$41°_{3}-140_{4}$	4	115	(62)	—	(1.331)	1.216
4115.374	40	$a^{4}F_{1}-z^{4}F°_{1}$	6	450	(675)	629	.406	.856
4113.726	30	$19°_{3}-116_{3}$	6	—	(67)	884	.880	.899
4111.931	8	$43°_{4}-114_{4}$	6	81	—	—	(1.154)	1.073
4110.381	35	$a^{2}F_{3}-z^{4}F°_{3}$	5	180	(90)	1539	.909	1.089
4106.134	30	$38°_{4}-138_{3}$	5	152	(77)	1719	1.192	1.040
4101.772	35	$a^{4}H_{5}-z^{2}I°_{6}$	6	118	(759)	1170	1.228	1.110
4098.981	15	$36°_{4}-136_{3}$	5	164	(78)	1809	1.233	1.069
4093.955	20	$22°_{3}-121_{3}$	5	138	(71)	1313	.969	.831
4092.715	18	$38°_{4}-140_{4}$	6	33	(149)	—	(1.188)	1.221
4090.947	6	$46°_{4}-149_{4}$	6	—	(—)	1211	1.216	(1.205)
4083.233	35	$35°_{4}-137_{4}$	6	50	(272)	—	1.082	1.130
4081.222	40	$a^{4}H_{5}-z^{4}G°_{5}$	6	237	(833)	788	.670	.907
4078.321	15	$a^{4}F_{2}-y^{4}G°_{3}$	5	124	(62)	1430	.990	1.114
4072.917	20	$11°_{3}-107_{3}$	6	188	(653)	916	.831	1.019
4068.836	25	$36°_{4}-140_{4}$	6	—	(59)	1227	1.234	1.220
4067.279	25	$a^{4}F_{3}-z^{4}F°_{3}$	6	128	(538)	1156	1.220	1.092
4062.223	40	$a^{2}G_{5}-z^{2}G°_{5}$	6	92	(322)	965	1.014	.922
4054.991	12	$9°_{4}-108_{3}$	5	546	(273)	1883	.517	1.062
4053.506	40	$1°_{3}-101_{4}$	5	136	(68)	1411	.801	.937
4042.584	50	$19°_{3}-122_{4}$	5	121	(59)	1433	.889	1.010
4040.762	70	$15°_{4}-117_{4}$	6	91	(411)	940	.986	.895
4037.665	25	$40°_{3}-144_{4}$	4	66	(33)	854	1.150	1.085
4028.411	35	$7°_{4}-109_{3}$	6	85	(476)	1000	.958	1.042
4025.150	12	$14°_{3}-119_{3}$	6	—	—	—	1.263	.387
4017.596	10a	$38°_{4}-144_{4}$	6	110	(490)	—	(1.188)	1.078
4015.877	20	$59°_{3}-177_{4}$	4	166	(79)	—	1.339	1.173
4014.899	60	$22°_{3}-125_{4}$	5	163	(81)	1696	.963	1.126
4007.589	15	$28°_{4}-130_{4}$	5	249	(125)	826	.702	.452
3992.913	15	$39°_{4}-146_{4}$	6	195	289	1146	1.241	1.046
3992.386	50	$15°_{4}-120_{3}$	5	88	(35)	1479	.995	1.083
3986.395	15	$41_{3}-148_{2}$	5	304	(147)	—	1.337	1.033
3984.675	40	$54°_{4}-170_{3}$	7b	—	(0)	945	(.955)	.953
3971.684	35	$19°_{3}-125_{4}$	5	246	(125)	1994	.887	1.134
3967.048	35	$10°_{2}-113_{4}$	5	119	(61)	1166	.957	.838
3964.503	25	$9°_{4}-112_{3}$	5	366	—	1062	.502	.136
3960.914	40	$9°_{4}-113_{4}$	6	319	478	673	.514	.833
3953.660	12	$19°_{3}-126_{2}$	4	165	83	491	.887	1.052
3947.973	20	$13°_{2}-121_{2}$	6	169	(420)	903	.986	.817
3946.681	20	$55°_{4}-175_{2}$	7b	—	(0)	1108	1.088	1.096
3940.972	10	$13°_{2}-123_{3}$	5	128	(64)	—	.982	1.110
3931.088	35	$3°_{4}-105_{4}$	6	95	(426)	—	(.854)	.949
3919.813	45	$35°_{3}-151_{6}$	5	105	57	1390	1.101	1.206
3914.949	18	$17°_{4}-126_{3}$	5	184	(90)	793	1.253	1.069
3909.313	35	$15°_{4}-125_{4}$	6	138	(626)	1010	.987	1.125

Wave-Length	Int.	Combination	Type	Sep.	p	n	g_1	g_2
3908.408	30	$47°_{4}-166_{3}$	7b	—	(0)	1216	(1.221)	1.222
3907.289	35	$62°_{3}-194_{3}$	7b	—	(0)	1073	1.077	1.076
3895.804	35	$25°_{2}-138_{3}$	5	308	(153)	1771	.728	1.036
3890.761	8	$5°_{3}-113_{3}$	4	158	(479)	750	.987	.829
3882.446	5	$9°_{4}-119_{3}$	5	123	(26)	577	.516	.393
3881.665	5	$12°_{3}-120_{3}$	6	—	(233)	1110	1.131	1.089
3878.372	15	$3°_{4}-107_{3}$	4	152	(70)	321	.855	1.007
3876.974	15	$24°_{3}-140_{4}$	5	88	(43)	—	(1.132)	1.220
3873.255	4	$65°_{2}-210_{2}$	5	141	70	—	(.939)	1.080
3868.502	3	$37°_{3}-155_{3}$	6	146	—	1666	1.012	1.158
3866.819	12	$2°_{4}-105_{4}$	7b	—	—	952	.952	.952
3862.465	15	$36°_{4}-155_{3}$	5	69	(0)	—	(1.243)	1.174
3853.164	25	$1°_{3}-104_{3}$	6	140	484	875	.794	.934
3838.542	35	$11°_{3}-121_{3}$	5	—	(0)	840	(.829)	.825
3836.112	15	$4°_{3}-115_{4}$	5	159	78	1225	.806	.965
3831.550	4	$27°_{2}-145_{3}$	4	—	(0)	1012	(1.022)	1.019
3823.903	8	$8°_{4}-122_{4}$	6	—	(68)	1.010	1.018	1.002
3821.700	6	$22°_{3}-141_{2}$	5	132	(68)	1296	.966	.834
3817.455	25	$28°_{3}-147_{3}$	4	128	(60)	—	.564	.692
3809.224	25	$29°_{4}-151_{6}$	5	102	(46)	1769	1.103	1.205
3803.097	35	$12°_{3}-125_{4}$	7b	—	(0)	1122	1.122	1.122
3799.038	4	$15°_{4}-133_{4}$	6	86	(384)	—	(.988)	1.074
3796.674	6	$54°_{4}-187_{4}$	6	142	(631)	1015	.955	1.097
3792.326	20	$16°_{3}-136_{3}$	7b	—	(0)	1082	1.082	1.082
3788.753	15	$18°_{3}-139_{6}$	6	—	(87)	1122	1.128	1.115
3781.620	25	$22°_{3}-144_{4}$	5	110	(59)	1473	.972	1.082
3777.672	12	$4°_{3}-119_{2}$	4	96	(43)	570	.810	.906
3771.606	15	$23°_{2}-145_{3}$	5	260	130	—	(.766)	1.018
3765.044	12	$26°_{4}-149_{4}$	6	226	—	1093	.977	1.203
3764.117	18	$12°_{4}-128_{3}$	6	—	(445)	1079	1.119	1.039
3760.694	6	$26°_{3}-148_{2}$	5	109	(—)	—	(1.132)	1.023
3757.862	15	$7°_{3}-125_{4}$	5	162	(82)	—	(.968)	1.130
3755.425	20	$13°_{2}-134_{3}$	5	212	(105)	—	.995	.783
3752.353	3	$36°_{4}-164_{3}$	5	108	(54)	—	(1.243)	1.135
3746.374	8	$26°_{4}-153_{3}$	4	145	—	—	.977	1.122
3741.727	10	$51°_{3}-186_{3}$	6	—	(92)	1169	1.156	1.182
3737.516	5	$24°_{3}-153_{3}$	7b	—	(0)	1126	1.126	1.126
3728.423	50	$34°_{4}-163_{2}$	7b	—	(0)	1198	1.198	1.198
3725.675	40	$42°_{3}-170_{3}$	7b	—	(0)	952	.952	.952
3722.750	12	$9°_{4}-129_{4}$	6	192	(288)	608	.512	.704
3722.098	10	$a^{4}H_{5}-z^{2}G°_{4}$	7b	—	(0)	119	(1.124)	1.125
3719.797	15	$7°_{3}-128_{3}$	6	—	(378)	1007	.971	1.042
3716.930	10	$51°_{3}-188_{3}$	7b	—	(0)	1154	1.154	1.154
3716.365	15	$1°_{3}-106_{3}$	6	42	149	808	.795	.837
3709.933	25	$2°_{4}-115_{4}$	6	—	(81)	957	.948	.966
3709.286	25	$21°_{3}-151_{6}$	6	—	(83)	1196	1.196	1.202
3704.977	8	$38°_{4}-170_{3}$	4	240	(120)	—	1.189	.947
3694.911	15	$6°_{4}-128_{3}$	7b	—	(0)	1041	1.041	1.041
3693.422	10	$66°_{3}-223_{2}$	7b	—	(0)	1196	(1.195)	1.195
3689.161	8	$24°_{3}-155_{3}$	6	—	(94)	1155	1.142	1.168
3687.802	10	$15°_{4}-144_{4}$	6	88	(398)	—	.992	(1.080)
3681.376	10	$59°_{3}-207_{3}$	6	148	(506)	—	1.345	(1.197)
3679.424	12	$57°_{2}-205_{3}$	4	154	76	730	1.271	1.117
3672.789	15	$52°_{3}-191_{6}$	6	—	(—)	1091	1.116	(1.066)
3668.719	12	$1°_{3}-108_{2}$	4	266	(135)	144	.800	1.066
3667.981	80	$12°_{3}-137_{3}$	6	—	(101)	1138	1.129	1.147
3662.990	8	$22°_{3}-155_{3}$	6	196	610	1072	.968	1.161
3659.227	20	$3°_{4}-123_{3}$	4	262	132	—	.857	1.119
3658.258	10	$8°_{4}-133_{4}$	6	—	213	1044	1.020	1.068
3656.755	10	$46°_{4}-186_{3}$	5	—	(0)	1221	1.196	(1.189)
3655.851	25	$7°_{3}-133_{4}$	4	110	(52)	487	.979	1.089
3653.108	15	$12°_{3}-140_{4}$	4	98	(53)	676	1.119	1.217
3651.651	5	$66°_{3}-226_{2}$	5	89	—	—	(1.195)	1.106
3647.954	10	$59°_{3}-209_{4}$	4	148	(75)	—	1.345	1.197
3641.537	10	$14°_{3}-146_{4}$	4	202	(101)	939	1.242	1.040
3638.275	10	$38°_{4}-177_{4}$	7b	—	(0)	1184	1.184	1.184
3636.371	3	$69°_{3}-238_{2}$	5	162	—	—	(1.123)	1.285
3632.106	10	$11°_{3}-138_{3}$	6	205	(720)	933	.825	1.035

TABLE I.—*Continued.*

Wave-Length	Int.	Combination	Type	Sep.	p	n	g_1	g_2
3630.421	6	$65°_2 - 226_2$	6	176	(427)	1022	.934	1.110
3628.247	10	$8°_4 - 137_2$	5	124	(59)	1711	1.032	1.156
3622.145	15	$47°_6 - 191_6$	6	130	(845)	1152	1.222	1.092
3613.701	18	$8°_4 - 140_4$	6	202	(919)	1130	1.023	1.225
3607.625	15	$33°_5 - 175_2$	4	100	(47)	745	1.000	1.100
3603.358	6	$31°_3 - 171_3$	4	137	(—)	—	1.152	1.015
3599.974	10	$27°_3 - 167_1$	5	382	(191)	839	1.024	.642
3596.115	12	$59°_3 - 213_3$	4	157	(76)	630	1.343	1.186
3590.598	50	$42°_3 - 184_1$	4	138	(68)	348	.951	1.089
3580.562	2	$4°_3 - 131_3$	6	244	(863)	925	.805	1.049
3573.701	10	$15_3 - 156_1$	6	131	586	—	(.988)	1.119
3563.823	2	$22°_3 - 164_3$	6	154	(534)	1044	.967	1.121
3562.091	6	$21°_4 - 163_4$	7b	—	(0)	1192	1.192	1.192
3561.538	6	$a^4H_3 - y^4G°_3$	4	159	(75)	—	(.949)	1.108
3559.328	6	$51°_3 - 205_3$	6	—	(113)	1145	1.161	1.129
3558.706	8	$37°_3 - 183_2$	4	223	(113)	448	1.006	1.230
3554.993	25	$8°_4 - 144_3$	6	—	(243)	1050	1.023	1.077
3552.727	18	$7°_3 - 144_1$	4	109	(54)	477	.967	1.076
3551.664	10	$16°_2 - 158_2$	4	147	(84)	716	1.083	1.230
3547.001	15	$23°_3 - 167_1$	5	131	—	—	(.766)	.635
3546.190	20	$67°_3 - 238_2$	6	267	(925)	933	.803	1.070
3539.086	100	$8°_4 - 145_3$	6	—	(20)	1018	1.022	1.016
3537.437	10	$37°_5 - 186_3$	6	181	(616)	1091	1.013	1.194
3536.696	10	$66°_3 - 235_4$	4	103	49	722	1.200	1.097
3530.022	18	$6°_4 - 144_4$	6	—	(154)	1055	1.038	1.072
3524.073	8	$41°_3 - 192_3$	5	254	—	—	(1.331)	1.077
3520.522	30	$3°_4 - 132_3$	4	218	(112)	—	.842	1.060
3519.077	25	$10°_2 - 148_2$	6	—	(145)	985	.956	1.014
3507.945	12	$3°_4 - 133_4$	6	215	(982)	975	.859	1.074
3495.478	10	$10°_2 - 154_2$	7b	—	(0)	959	.959	.959
3493.724	18	$16°_2 - 164_2$	6	—	(87)	1099	1.086	1.112
3493.110	12	$24°_3 - 171_3$	6	121	(426)	1070	1.130	1.010
3488.553	35	$48°_4 - 208_3$	4	117	57	568	1.211	1.094
3477.452	15	$48°_4 - 209_4$	6	—	85	1228	1.218	1.237
3476.842	35	$67°_2 - 244_2$	4	136	60	1052	1.526	1.390
3474.216	15	$13_2 - 161_2$	7b	—	(0)	1008	1.008	1.008
3468.113	20	$23°_2 - 175_2$	6	344	(854)	937	.758	1.102
3463.218	3	$48°_4 - 210_3$	5	152	(66)	1751	1.220	1.068
3461.344	15	$22°_3 - 175_2$	4	138	(69)	617	.961	1.099
3446.721	15	$7_3 - 156_4$	4	150	80	289	.965	1.115
3442.955	18	$31°_2 - 188_3$	7b	—	(0)	1150	(1.154)	1.153
3442.380	25	$25°_2 - 180_2$	6	180	—	820	.731	.911
3436.304	15	$24°_3 - 180_2$	5	221	(115)	1706	1.142	.921
3433.091	25	$66°_3 - 244_3$	6	204	(714)	1298	1.196	1.400
3423.853	20	$63°_1 - 238_2$	4	418	214	641	1.693	1.275
3417.450	30	$36°_4 - 199_4$	6	—	172	1253	1.234	1.272
3416.556	20	$41°_3 - 203_2$	5	124	64	1647	1.337	1.213
3404.910	18	$4°_3 - 154_2$	4	164	(85)	404	.814	.978
3403.603	15	$20°_4 - 181_4$	6	143	(215)	607	.535	.679
3394.138	15	$26°_4 - 186_3$	4	213	—	—	(.976)	1.189
3392.784	10	$25°_3 - 186_3$	5	462	(227)	2340	.722	1.184
3390.515	20	$15°_4 - 174_3$	4	272	(135)	—	.988	1.260
3387.780	20	$37°_3 - 203_3$	4	184	(87)	553	1.010	1.194
3368.690	20	$11°_3 - 164_3$	6	286	(998)	—	.840	1.126
3364.821	15	$22°_3 - 186_3$	6	222	(783)	1074	.963	1.185
3364.345	18	$2°_4 - 144_3$	6	128	(509)	1009	.948	1.076
3360.541	35	$a^4I°_4 - y^4I°_3$	7b	—	(0)	1197	1.197	1.197
3357.215	30	$22°_3 - 187_4$	5	123	(61)	1539	.988	1.111
3356.410	25	$13°_3 - 175_2$	6	110	(278)	1054	.999	1.109
3352.283	25	$25°_3 - 192_3$	6	338	(858)	919	.733	1.070
3349.967	30	$37°_3 - 207_3$	6	185	(643)	—	1.009	1.194
3444.761	50	$22°_3 - 188_3$	6	185	(654)	1058	.970	1.155
3341.868	40	$25°_3 - 193_3$	6	269	(672)	867	.730	.999
3334.455	20	$a^4I_4 - y^4I°_4$	7b	—	(0)	1104	1.104	1.104
3333.901	8	$55°_1 - 227_4$	6	185	(278)	993	1.086	.901
3333.660	20	$38°_4 - 209_4$	7b	—	(0)	1193	1.193	1.193
3331.224	10	$23°_3 - 192_3$	6	308	(775)	922	.768	1.078
3325.329	25	$a^4I_4 - y^4I°_4$	6	—	(139)	735	.750	.720

Wave-Length	Int.	Combination	Type	Sep.	p	n	g_1	g_2
3323.294	10	$10°_2 - 169_2$	6	201	(498)	1057	.957	1.158
3320.940	10	$23°_2 - 193_3$	6	240	(590)	887	.757	.997
3317.797	15	$36°_4 - 209_4$	6	—	(166)	1226	1.244	1.208
3314.721	25	$19°_3 - 188_3$	6	273	(950)	1013	.871	1.144
3312.215	30	$a^4I_3 - y^4I°_3$	7b	—	(0)	964	.964	.964
3311.497	15	$13°_2 - 180_2$	6	—	(195)	952	.991	.913
3306.632	20	$42°_3 - 213_4$	4	230	(111)	—	.954	1.184
3304.836	30	$36°_4 - 210_3$	5	170	—	1825	1.240	1.070
3296.883	25	$45°_4 - 216_2$	5	167	(84)	1340	.924	1.091
3295.289	30	$19°_3 - 192_2$	4	190	(95)	404	.880	1.070
3293.41	20	$15°_4 - 186_3$	4	203	118	291	.989	1.192
3286.029	18	$25°_2 - 200_2$	6	404	(1005)	941	.740	1.144
3285.224	35	$19°_3 - 193_2$	4	120	(62)	602	.886	1.006
3276.251	18	$10°_2 - 175_2$	6	140	(362)	1022	.960	1.100
3272.253	40	$35°_3 - 213_4$	4	—	—	—	1.073	1.159
3271.151	18	$15°_4 - 188_3$	4	165	(85)	—	.986	1.151
3270.133	12	$2°_4 - 155_3$	4	219	—	188	.954	1.173
3259.784	20	$22°_3 - 200_2$	4	169	(79)	553	.978	1.147
3256.682	20	$6°_4 - 174_3$	4	217	(108)	277	1.033	1.250
3254.866	12	$25°_2 - 203_2$	6	496	(1169)	954	.729	1.225
3246.674	35	$13°_2 - 188_3$	5	153	(77)	1557	1.000	1.153
3243.370	10	$26°_4 - 205_3$	4	133	(68)	510	.975	1.108
3236.735	35	$24°_3 - 205_3$	7b	—	(0)	1127	1.127	1.127
3232.290	15	$41°_3 - 217_3$	6	272	(930)	1194	1.334	1.062
3231.236	30	$19°_3 - 200_2$	4	—	—	—	.879	1.129
3229.363	25	$9°_1 - 180_2$	5	400	(200)	1524	.527	.927
3227.114	25	$9°_1 - 181_1$	6	149	(222)	589	.516	.665
3225.672	25	$59°_3 - 244_3$	6	—	(169)	1381	1.356	1.404
3221.171	30	$26°_4 - 172_2$	4	215	(108)	215	.973	1.188
3218.380	30	$13°_2 - 193_3$	7b	—	(0)	990	.990	.990
3212.480	20	$55°_1 - 241_3$	6	305	(468)	1250	1.094	1.399
3199.279	25	$36°_4 - 218_4$	6	136	(613)	1170	1.238	1.102
3190.341	25	$11°_3 - 186_3$	6	360	(1252)	988	.812	1.172
3189.638	30	$10°_2 - 186_3$	5	236	(114)	1769	.958	1.191
3188.787	25	$19°_3 - 205_3$	6	236	(831)	997	.881	1.117
3183.523	40	$26°_4 - 210_3$	4	100	—	—	.987	1.087
3177.137	20	$24°_3 - 210_3$	6	—	(197)	1100	1.128	1.072
3164.154	40	$6°_4 - 186_3$	4	142	(71)	528	1.034	1.176
3161.026	15	$37°_3 - 223_3$	4	—	—	—	1.011	1.190
3155.704	20	$26°_4 - 213_4$	6	211	—	—	.978	1.189
3154.506	15	$11°_3 - 192_3$	4	242	(120)	226	.826	1.068
3145.283	30	$11°_3 - 193_3$	4	162	81	—	.815	.977
3144.596	25	$10°_2 - 193_3$	6	—	(82)	969	.952	.986
3142.312	25	$2°_4 - 170_3$	7b	—	(0)	947	.947	.947
3130.334	30	$22°_3 - 213_4$	5	226	—	1980	.958	1.184
3127.530	40	$15°_4 - 207_3$	4	208	(107)	262	.991	1.199
3110.278	30	$19°_3 - 212_3$	4	176	(90)	435	.876	1.052
3103.377	15	$15°_4 - 209_4$	6	214	(953)	—	.989	1.203
3082.304	20	$24°_3 - 217_3$	6	—	(233)	1103	1.136	1.070
3078.908	8	$31°_2 - 225_3$	7b	—	(0)	1142	(1.154)	1.151
3072.886	20	$31°_2 - 226_2$	6	—	—	1135	1.154	1.114
3071.110	20	$4°_3 - 193_2$	4	175		360	.803	.978
3064.024	15	$22°_3 - 217_3$	6	—	318	1006	.961	1.051
3044.395	15	$30°_2 - 228_2$	6	—	123	1064	1.039	1.089
3028.958	12	$43°_4 - 244_3$	4	238	123	—	1.153	1.391
3026.620	18	$20°_1 - 222_1$	6	—	154	574	.522	.625
3017.195	20	$36°_4 - 237_3$	7b	—	—	1247	(1.241)	1.239
3008.789	40	$8°_4 - 209_4$	6	178	(800)	1112	1.024	1.202
3003.562	12	$11°_3 - 210_3$	6	244	(852)	959	.828	1.072
3002.748	20	$25°_2 - 227_1$	4	174	(87)	458	.730	.904
2995.644	30	$33°_3 - 237_3$	6	241	(846)	—	.996	1.237
2994.418	12	$22°_3 - 226_2$	4	135	(—)	620	.956	1.091
2990.873	40	$6°_4 - 209_4$	6	156	(710)	1130	1.046	1.202
2983.918	6	$10°_2 - 212_2$	6	—	(126)	992	.967	1.017
2981.905	15	$23°_2 - 228_2$	6	328	(816)	930	.760	1.088
2976.905	8	$22°_3 - 228_2$	4	109	—	676	.949	1.058
2972.582	5	$59°_3 - 259_2$	5	190	(88)	1250	1.352	1.162
2970.316	10	$19°_2 - 226_2$	4	219	(106)	322	.880	1.091

TABLE I.—*Continued.*

Wave-length	Int.	Combination	Type	Sep.	p	n	g_1	g_2
2965.269	15	$31°_{2\frac{1}{2}}-238_{2\frac{1}{2}}$	6	—	(284)	1224	1.167	1.281
2964.801	25	$36°_{4\frac{1}{2}}-244_{2\frac{1}{2}}$	4	150	(67)	703	1.227	1.377
2960.438	10	$61°_{2\frac{1}{2}}-260_{3\frac{1}{2}}$	7*b*	—	(0)	1158	1.158	1.158
2946.382	8	$55°_{1\frac{1}{2}}-257_{1\frac{1}{2}}$	6	206	—	991	1.096	.888
2940.663	4	$52°_{1\frac{1}{2}}-256_{2\frac{1}{2}}$	7*b*	—	(0)	1090	(1.104)	1.107
2918.665	30	$11°_{2\frac{1}{2}}-217_{2\frac{1}{2}}$	6	230	(804)	951	.836	1.066
2912.906	12	$66°_{2\frac{1}{2}}-262_{2\frac{1}{2}}$	4	354	(173)	336	1.219	1.573
2908.421	30	$24°_{2\frac{1}{2}}-238_{2\frac{1}{2}}$	4	126	(61)	817	1.132	1.258
2839.364	18	$9°_{1\frac{1}{2}}-227_{1\frac{1}{2}}$	6	401	(603)	711	.513	.914
2837.896	10	$45°_{1\frac{1}{2}}-257_{1\frac{1}{2}}$	7*b*	—	(0)	903	.903	.903
2830.899	30	$51°_{2\frac{1}{2}}-260_{3\frac{1}{2}}$	7*b*	—	(0)	1143	1.143	1.143
2729.162	8	$40°_{2\frac{1}{2}}-260_{3\frac{1}{2}}$	7*b*	—	(0)	1136	1.136	1.136
2708.131	2	$31°_{2\frac{1}{2}}-259_{2\frac{1}{2}}$	7*b*	—	(0)	1160	1.160	1.160
2598.250	5	$6°_{4\frac{1}{2}}-252_{3\frac{1}{2}}$	7*b*	—	(0)	1038	(1.040)	1.039
2592.338	10	$10°_{2\frac{1}{2}}-255_{2\frac{1}{2}}$	7*b*	—	(0)	946	.946	.946

as to make possible intercomparison in cases of asymmetry.

The data reported are taken from exposures made at the following fields:

Set No.	*Field*	*Set No.*	*Field*
Z-24*a*	92,240 oersteds	*Z*-40*b*	73,300 oersteds
Z-24*b*	72,180	*Z*-52	87,600
Z-25	91,900	*Z*-54	87,550
Z-40*a*	96,900		

The field intensities were calculated from *raies ultimes* of Ag, Ca and Cu, which were measured several times by different observers. The current through the electromagnet, and hence the field, was held constant to within ± 0.1 percent during exposures, which usually did not exceed 40 min. in duration.

All spectra were measured on an automatic comparator.[7] The earlier plates were measured in both directions, but inasmuch as the results from the two directions were found to give line separations which agreed to within 0.001A in most cases, the reverse run was considered to be unnecessary and was omitted on the later plates. Patterns for most lines were measured at least twice at each of three fields, and the final g values were usually found consistent from the various plates to within ± 0.003 unit.

Reduction of Plates

Zeeman patterns of 180 resolved lines were first reduced by the junior author.[8] Values of g precise, on the average, to 5 units in the third figure after the decimal point were obtained for 128 levels, the preliminary classification[1] was verified, and four assignments whose agreements were fortuitous were corrected. These Zeeman data proved to be of great value in the extension of the classification, so further plates were taken at several field intensities. These have been measured and reduced with the assistance of W.P.A. clerical help. Three completely independent sets of determinations were made from the best plates, so that errors of interpretation or of calculation could be checked. Most of the g-values included in Table II agreed in the several determinations to within 3 units in the third figure after the decimal. In cases where this was not originally true, new measurements were made.

The measurements reported here differ from most previous Zeeman determinations in the high field intensities used, and in the complexity of the spectrum studied. In cases where a strong isolated line was measured, single determinations would give g values reproducible to within 1 or 2 units in the third figure after the decimal. The cerium spectrum is so rich, however, that many patterns overlap, and in such cases the patterns are difficult to disentangle. About five times as many lines have been studied as are included in Table I, the present paper dealing only with those giving results precise enough to be of value in studying magnetic interactions in the atom.

Results

In Table I are given data for 427 lines of Ce II for which Zeeman patterns have been completely resolved or are otherwise such as to give definite J and g values. To have listed the positions of all components of each pattern would have required an undue amount of space, yet we have been convinced of the desirability of including enough of the raw material of measurement to permit

[7] G. R. Harrison, J. Opt. Soc. Am. **25**, 169 (1935); Rev. Sci. Inst. **9**, 15 (1938); G. R. Harrison and J. P. Molnar, J. Opt. Soc. Am. **33**, 43 (1940).

[8] N. F. Hosford, unpublished M.S. Thesis, M. I. T., 1939.

TABLE II.

Term	Term Value	g Values LS	g Values Obs.	No. Lines
Low levels—Group 1.				
$a^4H_{3\frac{1}{2}}$	0.00	0.667	0.669 ±0.001	5
$4\frac{1}{2}$	311.55	0.970	0.949	1 5
$5\frac{1}{2}$	1659.70	1.132	1.125	1 6
$6\frac{1}{2}$	3113.56	1.231	1.225	3 5
$a^2H_{4\frac{1}{2}}$	1762.72	0.909	0.934	1 5
$5\frac{1}{2}$	3487.01	1.091	1.089	1 2
$b^2H_{4\frac{1}{2}}$	3157.84	0.909	0.889	2 3
$5\frac{1}{2}$	4424.06	1.091	0.957	— 1
$a^4K_{4\frac{1}{2}}$	3238.32	0.770	0.803	1 1
$6\frac{1}{2}$	4569.70	0.964	0.980	2 2
$7\frac{1}{2}$	5917.95	1.090	1.090	1 1
$8\frac{1}{2}$	7311.75	1.176	1.187	— 1
$a^4F_{1\frac{1}{2}}$	3600.92	0.400	0.407	2 3
$2\frac{1}{2}$	3868.26	1.028	0.986	1 5
$3\frac{1}{2}$	4677.68	1.238	1.213	1 3
$4\frac{1}{2}$	4920.04	1.333	1.247	1 2
$a^4I_{4\frac{1}{2}}$	4277.26	0.728	0.751	5 2
$5\frac{1}{2}$	5042.81	0.965	0.961	5 2
$6\frac{1}{2}$	6204.27	1.107	1.105	1 3
$7\frac{1}{2}$	7456.03	1.200	1.193	3 2
$a^4G_{2\frac{1}{2}}$	4594.64	0.572	0.633	— 1
$3\frac{1}{2}$	5462.89	0.986	0.992	— 2
$4\frac{1}{2}$	6460.10	1.172	1.04	— 1
$5\frac{1}{2}$	7604.40	1.272	1.235	— 1
$a^2F_{3\frac{1}{2}}$	4935.37	0.857	0.905	1 2
$3\frac{1}{2}$	5199.66	1.142	1.026	1 2
$b^4H_{3\frac{1}{2}}$	5871.75	0.667	0.694	1 1
$4\frac{1}{2}$	6849.53	0.970	0.969	10 2
$5\frac{1}{2}$	7905.49	1.132	1.124	2 2
$6\frac{1}{2}$	9173.76	1.231	1.210	10 1
$a^2I_{5\frac{1}{2}}$	6792.03	0.924	0.951	40 1
$6\frac{1}{2}$	7600.67	1.077	1.101	1 1
$a^2G_{4\frac{1}{2}}$	7015.58	1.111	1.193	— 1
$5\frac{1}{2}$	7161.62	0.889	1.016	3 2
$c^4H_{2\frac{1}{2}}$	8602.79	0.667	0.691	1 2
$4\frac{1}{2}$	9673.21	0.970	0.963	2 1
$5\frac{1}{2}$	10422.27	1.132	1.114	— 3
$6\frac{1}{2}$	11427.60	1.231	1.204	1 3
$b^4F_{1\frac{1}{2}}$	8850.59	0.400	0.467	1 1
Middle levels—Group 1.				
$z^4I°_{4\frac{1}{2}}$	21912.36	0.728	0.799 ±0.002	4
$5\frac{1}{2}$	23524.55	0.965	1.005	1 5
$6\frac{1}{2}$	25189.86	1.107	1.130	1 4
$7\frac{1}{2}$	26992.60	1.200	1.205	1 4
$z^4H°_{3\frac{1}{2}}$	22414.19	0.667	0.712	1 3
$4\frac{1}{2}$	23578.78	0.970	0.941	2 5
$5\frac{1}{2}$	24473.07	1.132	1.036	15 3
$6\frac{1}{2}$	25737.94	1.231	1.158	5 3
$z^2H°_{4\frac{1}{2}}$	24121.63	0.909	0.933	2 6
$5\frac{1}{2}$	25409.35	1.091	1.107	2 4
$z^4G°_{2\frac{1}{2}}$	24542.14	0.572	0.594	1 3
$3\frac{1}{2}$	24495.58	0.986	0.900	5 1
$4\frac{1}{2}$	25881.44	1.172	1.112	3 1
$5\frac{1}{2}$	27297.55	1.272	1.210	2 2
$z^2I°_{5\frac{1}{2}}$	25896 54	0.924	1.019	1 1
$6\frac{1}{2}$	27486.38	1.077	1.110	1 1
$z^2G°_{3\frac{1}{2}}$	26038.65	0.889	0.940	40 1
$4\frac{1}{2}$	26815.67	1.111	1.110	9 4
$y^4G°_{2\frac{1}{2}}$	26094.89	0.572	0.612	5 3
$3\frac{1}{2}$	28381.22	0.986	1.111	2 3
$4\frac{1}{2}$	27180.09	1.172	—	— —
$5\frac{1}{2}$	27490.99	1.272	—	— —
$z^4D°_{1\frac{1}{2}}$	26266.39	1.200	0.677	40 1
$2\frac{1}{2}$	29031.38	1.372	1.102	3 1

Term	Term Value	g Values LS	g Values Obs.	No. Lines
Middle levels—Group 1—Cont.				
$y^2G°_{3\frac{1}{2}}$	27277.01	0.889	0.975 ±0.003	1
$4\frac{1}{2}$	28518.61	1.111	1.104	10 2
$z^4F°_{1\frac{1}{2}}$	27893.22	0.400	0.860	6 2
$2\frac{1}{2}$	28112.76	1.028	1.015	— 1
$3\frac{1}{2}$	29257.16	1.238	1.086	3 3
$4\frac{1}{2}$	29186.35	1.333	1.106	6 2
$z^2F°_{3\frac{1}{2}}$	29135.26	1.142	1.10	— 1
$y^2F°_{3\frac{1}{2}}$	29681.66	1.142	0.966	— 1
$x^2G°_{4\frac{1}{2}}$	30054.23	1.111	1.11	— 1
$3\frac{1}{2}$	31771.76	0.889	0.923	— 1
$y^2H°_{4\frac{1}{2}}$	31172.98	0.909	1.142	— 1
$5\frac{1}{2}$	31298.21	1.091	—	— —
$y^4I°_{4\frac{1}{2}}$	34340.86	0.728	0.720	— 1
$5\frac{1}{2}$	35225.35	0.965	0.964	— 1
$6\frac{1}{2}$	36185.59	1.107	1.104	— 1
$7\frac{1}{2}$	37204.60	1.200	1.197	— 1

Term	Desig.	Term Value	g Values Obs.	No. Lines
Low levels—Group II				
$^4H°_{2\frac{1}{2}}$	1°	0.00	0.794 ±0.002	4
$^4H°_{4\frac{1}{2}}$	2°	987.62	0.948	1 4
$^4I°_{4\frac{1}{2}}$	3°	1410.30	0.856	2 4
$^4H°_{3\frac{1}{2}}$	4°	1873.95	0.806	1 6
$^4D°_{\frac{1}{2}}$	5°	2140.50	0.985	3 2
$^4H°_{4\frac{1}{2}}$	6°	2382.26	1.039	2 8
$^4I°_{5\frac{1}{2}}$	7°	2563.26	0.968	2 5
$^2G°_{4\frac{1}{2}}$	8°	2581.26	1.023	1 10
$^4F°_{1\frac{1}{2}}$	9°	2595.65	0.516	2 9
$2\frac{1}{2}$	10°	2634.68	0.957	1 11
$3\frac{1}{2}$	11°	2641.57	0.829	1 5
$^4H°_{2\frac{1}{2}}$	12°	2879.69	1.123	2 5
$2\frac{1}{2}$	13°	3363.43	0.995	5 7
$\frac{1}{2}$	14°	3508.51	1.262	2 1
$4\frac{1}{2}$	15°	3593.89	0.988	2 11
$3\frac{1}{2}$	16°	3703.61	1.085	1 5
$^4D°_{1\frac{1}{2}}$	17°	3745.50	1.240	8 2
$^4I°_{6\frac{1}{2}}$	18°	3793.63	1.128	8 1
$2\frac{1}{2}$	19°	3995.47	0.879	3 14
$1\frac{1}{2}$	20°	4201.93	0.528	4 2
$^4H°_{6\frac{1}{2}}$	21°	4203.96	1.189	2 2
$2\frac{1}{2}$	22°	4266.40	0.964	5 11
$2\frac{1}{2}$	23°	4322.71	0.766	1 6
$2\frac{1}{2}$	24°	4459.88	1.134	2 12
$2\frac{1}{2}$	25°	4511.28	0.731	2 8
$^4H°_{4\frac{1}{2}}$	26°	4523.03	0.978	2 7
$2\frac{1}{2}$	27°	4737.37	1.021	3 2
$1\frac{1}{2}$	28°	4844.63	0.699	6 3
$^2H°_{3\frac{1}{2}}$	29°	4910.93	1.095	3 2
$2\frac{1}{2}$	30°	5010.88	1.038	4 2
$2\frac{1}{2}$	31°	5118.82	1.152	2 6
$\frac{1}{2}$	32°	5283.05	0.566	— 1
$3\frac{1}{2}$	33°	5437.44	0.999	1 3
$^4I°_{7\frac{1}{2}}$	34°	5455.92	1.196	2 1
$^4H°_{5\frac{1}{2}}$	35°	5651.37	1.085	4 4
$4\frac{1}{2}$	36°	5675.77	1.241	2 10
$3\frac{1}{2}$	37°	5716.23	1.009	2 6
$4\frac{1}{2}$	38°	5819.12	1.189	2 6
$1\frac{1}{2}$	39°	5924.22	1.241	3 2
$3\frac{1}{2}$	40°	5942.79	1.145	4 2
$3\frac{1}{2}$	41°	5964.98	1.332	2 3
$^2I°_{5\frac{1}{2}}$	42°	5969.01	0.954	2 3
$4\frac{1}{2}$	43°	6389.93	1.154	— 1

Term	Desig.	Term Value	g Values Obs.	No. Lines
Low levels—Group II—Cont.				
$^2I°_{5\frac{1}{2}}$	44°	6517.62	0.954 ±0.020	2
$1\frac{1}{2}$	45°	6521.33	0.924	15 2
$4\frac{1}{2}$	46°	6638.25	1.215	4 2
$^4H°_{6\frac{1}{2}}$	47°	6913.41	1.217	3 4
$6\frac{1}{2}$	48°	7059.09	1.222	2 3
$2\frac{1}{2}$	49°	7202.54	1.106	— 1
$^4G°_{5\frac{1}{2}}$	50°	7233.65	1.266	1 2
$^4G°_{3\frac{1}{2}}$	51°	7259.08	1.140	12 8
$^2I°_{6\frac{1}{2}}$	52°	7293.96	1.100	6 4
$5\frac{1}{2}$	53°	7522.65	1.248	3 3
$6\frac{1}{2}$	54°	7713.10	0.950	4 3
$1\frac{1}{2}$	55°	7818.18	1.099	7 4
$2\frac{1}{2}$	56°	7878.35	1.195	— 1
$2\frac{1}{2}$	57°	8175.90	1.271	4 5
$2\frac{1}{2}$	58°	8280.96	1.189	6 2
$3\frac{1}{2}$	59°	8402.68	1.343	3 5
$(2\frac{1}{2}, 2\frac{1}{2})$	60°	8694.57	—	— —
$3\frac{1}{2}$	61°	8804.27	1.160	7 2
$3\frac{1}{2}$	62°	8927.54	1.071	5 3
$1\frac{1}{2}$	63°	9634.22	1.690	3 3
$4\frac{1}{2}$	64°	9723.34	1.348	— 1
$2\frac{1}{2}$	65°	10114.90	0.938	4 3
$3\frac{1}{2}$	66°	10275.01	1.195	1 4
$2\frac{1}{2}$	67°	10641.48	1.526	— 1
$1\frac{1}{2}$	68°	10798.60	0.971	— 1
$3\frac{1}{2}$	69°	11340.59	1.123	1 2
$3\frac{1}{2}$	70°	11742.56	1.081	3 1
$3\frac{1}{2}$	71°	11949.19	1.029	3 1
$3\frac{1}{2}$	72°	12260.09	1.168	0 2
$(4\frac{1}{2}, 2\frac{1}{2})$	73°	12365.78	0.921	1 1
$4\frac{1}{2}$	74°	12762.67	1.275	— 1
$\frac{1}{2}$	75°	13311.30	—	— —

Term	Desig.	Term Value	g Values Obs.	No. Lines
Middle levels—Group II				
$^4I_{4\frac{1}{2}}$	101	24663.05	0.933 ±0.003	5
$^4G_{2\frac{1}{2}}$	102	25359.69	0.761	2 4
$1\frac{1}{2}$	103	25681.50	0.692	2 4
$3\frac{1}{2}$	104	25945.41	0.941	5 6
$4\frac{1}{2}$	105	26841.40	0.954	5 3
$^4H_{2\frac{1}{2}}$	106	26900.36	0.841	2 6
$2\frac{1}{2}$	107	27187.05	1.015	3 5
$2\frac{1}{2}$	108	27249.68	1.061	2 3
$^4I_{5\frac{1}{2}}$	109	27379.95	1.052	6 2
$5\frac{1}{2}$	110	27514.68	—	— —
$^4H_{3\frac{1}{2}}$	111	27811.52	P.B.	— —
$2\frac{1}{2}$	112	27812.41	P.B.	— —
$1\frac{1}{2}$	113	27835.23	0.833	3 4
$6\frac{1}{2}$	114	27905.18	0.920	— 1
$^4H_{4\frac{1}{2}}$	115	27934.65	0.969	3 4
$^4G_{3\frac{1}{2}}$	116	28297.49	0.906	3 3
$4\frac{1}{2}$	117	28334.76	0.891	3 3
$2\frac{1}{2}$	118	28337.81	0.916	— 1
$\frac{1}{2}$	119	28345.34	0.387	2 2
$4\frac{1}{2}$	120	28634.51	1.080	2 4
$2\frac{1}{2}$	121	28685.77	0.827	— 1
$4\frac{1}{2}$	122	28725.16	1.002	4 3
$3\frac{1}{2}$	123	28730.75	1.124	4 3
$1\frac{1}{2}$	124	29029.37	0.980	2 1
$^4G_{4\frac{1}{2}}$	125	29166.61	1.124	3 6
$^4G_{2\frac{1}{2}}$	126	29281.37	1.052	4 3
$3\frac{1}{2}$	127	29364.76	0.894	— 2

TABLE II.—*Continued.*

TERM	DESIG.	TERM VALUE	g VALUES OBS.		NO. LINES
Middle levels—Group II—Cont.					
$^4G_{3\frac{1}{2}}$	128	29438.83	1.041 ±0.001		3
$4\frac{1}{2}$	129	29449.82	0.723	—	—
$\frac{1}{2}$	130	29790.26	0.45	—	—
$3\frac{1}{2}$	131	29794.53	1.052	4	3
$3\frac{1}{2}$	132	29807.09	1.055	—	1
$^4H_{4\frac{1}{2}}$	133	29908.91	1.074	5	4
$1\frac{1}{2}$	134	29984.08	0.773	10	1
$2\frac{1}{2}$	135	29994.05	—	—	—
$3\frac{1}{2}$	136	30065.19	1.075	5	2
$^4H_{2\frac{1}{2}}$	137	30134.92	1.146	7	3
$3\frac{1}{2}$	138	30166.08	1.036	1	2
$^4I_{6\frac{1}{2}}$	139	30180.11	1.115	3	3
$4\frac{1}{2}$	140	30245.89	1.225	3	6
$2\frac{1}{2}$	141	30425.37	0.831	1	—
$4\frac{1}{2}$	142	30576.84	—	—	—
$3\frac{1}{2}$	143	30637.18	1.125	—	1
$4\frac{1}{2}$	144	30702.63	1.079	3	10
$3\frac{1}{2}$	145	30829.16	1.016	2	2
$1\frac{1}{2}$	146	30961.54	1.043	3	2
$\frac{1}{2}$	147	31032.64	0.692	—	1
$2\frac{1}{2}$	148	31043.14	1.014	—	1
$4\frac{1}{2}$	149	31075.61	1.207	—	1
$^4G_{5\frac{1}{2}}$	150	31089.74	1.221	—	1
$^4H_{6\frac{1}{2}}$	151	31155.66	1.203	3	2
$1\frac{1}{2}$	152	31170.67	1.118	—	1
$3\frac{1}{2}$	153	31207.96	1.123	2	2
$2\frac{1}{2}$	154	31234.90	0.975	5	2
$3\frac{1}{2}$	155	31558.64	1.167	5	5
$4\frac{1}{2}$	156	31568.03	1.138	—	1
$^4H_{3\frac{1}{2}}$	157	31738.50	1.140	—	1
$2\frac{1}{2}$	158	31851.42	1.238	—	1
$4\frac{1}{2}$	159	31930.96	1.149	—	1
$3\frac{1}{2}$	160	31937.65	—	—	—
$2\frac{1}{2}$	161	32138.71	1.008	—	1
$4\frac{1}{2}$	162	32197.99	1.195	10	3
$^4I_{7\frac{1}{2}}$	163	32269.31	1.195	3	2
$3\frac{1}{2}$	164	32318.20	1.118	6	4
$4\frac{1}{2}$	165	32413.38	1.019	6	2
$5\frac{1}{2}$	166	32492.06	1.221	3	1
$1\frac{1}{2}$	167	32507.50	0.643	4	2
$1\frac{1}{2}$	168	32638.19	1.070	—	1
$2\frac{1}{2}$	169	32716.67	1.151	—	1
$5\frac{1}{2}$	170	32802.12	0.948	3	4
$3\frac{1}{2}$	171	32862.79	1.009	2	3
$2\frac{1}{2}$	172	33045.13	1.188	—	1
$^4I_{1\frac{1}{2}}$	173	33050.34	1.426 ±0.003		2
$3\frac{1}{2}$	174	33079.54	1.257	3	2
$2\frac{1}{2}$	175	33148.62	1.098	4	6
$\frac{1}{2}$	176	33177.21	1.484	—	1
$4\frac{1}{2}$	177	33296.84	1.173	—	—
$3\frac{1}{2}$	178	33409.90	—	—	—
$6\frac{1}{2}$	179	33531.42	1.257	—	1
$2\frac{1}{2}$	180	33552.59	0.920	4	4
$1\frac{1}{2}$	181	33574.17	0.675	3	2
$6\frac{1}{2}$	182	33659.99	1.249	3	3
$2\frac{1}{2}$	183	33808.31	1.230?	—	1
$4\frac{1}{2}$	184	33811.54	1.090	—	1
$1\frac{1}{2}$	185	33876.36	1.535	5	2
$3\frac{1}{2}$	186	33977.16	1.186	4	8
$4\frac{1}{2}$	187	34044.46	1.102	6	2
$3\frac{1}{2}$	188	34155.33	1.151	2	5
$2\frac{1}{2}$	189	34166.08	—	—	—
$3\frac{1}{2}$	190	34255.25	1.046	—	1
$4\frac{1}{2}$	191	34295.45	1.125	—	1
$2\frac{1}{2}$	192	34333.12	1.069	3	4
$2\frac{1}{2}$	193	34426.07	0.997	3	5
$6\frac{1}{2}$	194	34513.49	1.079	2	3
$2\frac{1}{2}$	195	34574.47	—	—	—
$1\frac{1}{2}$	196	34767.70	0.974	4	2
$2\frac{1}{2}$	197	34813.21	1.269	6	2
$3\frac{1}{2}$	198	34920.78	1.109	—	1
$4\frac{1}{2}$	199	34928.99	1.275	—	1
$2\frac{1}{2}$	200	34935.46	1.134	6	4
$4\frac{1}{2}$	201	35017.67	—	—	—
$1\frac{1}{2}$	202	35197.90	0.999	—	1
$2\frac{1}{2}$	203	35225.69	1.217	6	2
$2\frac{1}{2}$	204	35266.21	1.042	—	1
$3\frac{1}{2}$	205	35346.31	1.119	5	5
$1\frac{1}{2}$	206	35457.37	2.799?	—	1
$3\frac{1}{2}$	207	35558.70	1.197	2	5
$5\frac{1}{2}$	208	35716.09	1.096	—	1
$4\frac{1}{2}$	209	35807.57	1.199	3	6
$3\frac{1}{2}$	210	35925.70	1.075	3	5
$3\frac{1}{2}$	211	36112.04	1.184	—	1
$2\frac{1}{2}$	212	36137.62	1.060	20	2
$4\frac{1}{2}$	213	36202.54	1.186	2	4
$1\frac{1}{2}$	214	36516.85	—	—	—
$1\frac{1}{2}$	215	36686.79	—	—	—
$3\frac{1}{2}$	216	36844.35	1.091	—	1
$2\frac{1}{2}$	217	36893.83	1.062	6	4
$^4I_{4\frac{1}{2}}$	218	36923.81	1.104	—	—
$6\frac{1}{2}$	219	37055.67	—	—	—
$1\frac{1}{2}$	220	37078.89	0.695	0.038	2
$4\frac{1}{2}$	221	37196.17	1.175	2	2
$1\frac{1}{2}$	222	37232.48	0.625	—	1
$2\frac{1}{2}$	223	37342.37	1.194	3	2
$1\frac{1}{2}$	224	37530.68	1.112	—	1
$3\frac{1}{2}$	225	37588.53	1.142	—	1
$2\frac{1}{2}$	226	37652.13	1.108	4	4
$1\frac{1}{2}$	227	37804.48	0.908	3	3
$2\frac{1}{2}$	228	37848.58	1.082	5	2
$5\frac{1}{2}$	229	37970.63	—	—	—
$3\frac{1}{2}$	230	37973.83	—	—	—
$2\frac{1}{2}$	231	38134.45	0.621	—	1
$5\frac{1}{2}$	232	38137.78	—	—	—
$1\frac{1}{2}$	233	38452.83	—	—	—
$2\frac{1}{2}$	234	38529.08	1.169	8	3
$4\frac{1}{2}$	235	38541.94	1.097	1	3
$5\frac{1}{2}$	236	38741.70	—	—	—
$3\frac{1}{2}$	237	38809.52	1.192	2	2
$2\frac{1}{2}$	238	38832.75	1.273	5	4
$\frac{1}{2}$	239	38857.02	—	—	—
$4\frac{1}{2}$	240	38933.30	—	—	—
$1\frac{1}{2}$	241	38937.88	1.399	—	1
$4\frac{1}{2}$	242	38953.80	1.268	—	1
$2\frac{1}{2}$	243	39321.72	—	—	—
$3\frac{1}{2}$	244	39395.02	1.396	3	4
$3\frac{1}{2}$	245	39400.44	—	—	—
$6\frac{1}{2}$	246	39603.92	0.740	—	1
$2\frac{1}{2}$	247	39838.55	1.577	—	1
$4\frac{1}{2}$	248	39871.89	—	—	—
$1\frac{1}{2}$	249	39979.16	—	—	—
$1\frac{1}{2}$	250	40511.18	—	—	—
$5\frac{1}{2}$	251	40673.67	—	—	—
$3\frac{1}{2}$	252	40858.26	—	—	—
$4\frac{1}{2}$	253	41083.38	—	—	—
$3\frac{1}{2}$	254	41085.29	—	—	—
$2\frac{1}{2}$	255	41198.37	—	—	—
$5\frac{1}{2}$	256	41290.06	1.090	—	1
$1\frac{1}{2}$	257	41748.35	0.880	20	2
$6\frac{1}{2}$	258	41992.39	—	—	—
$2\frac{1}{2}$	259	42033.75	1.161	2	2
$3\frac{1}{2}$	260	42573.26	1.143	5	3
$2\frac{1}{2}$	261	43460.59	1.170	2	2
$2\frac{1}{2}$	262	44594.96	1.573	—	1

reconstruction of patterns and detailed checking of results. We have adopted a shorthand notation suggested by Meggers,[9] based on that of Back-Landé,[10] and modified by ourselves in the interest of convenience.

The columns of Table I, from left to right, contain, respectively, the wave-length in angstroms and the arc intensity of the line as given by the *M.I.T. Wavelength Tables*,[11] the lower and upper levels to which we have assigned the line, the type of Zeeman pattern observed (in accordance with the listing below); the directly observed fundamental pattern separation (g_1-g_2) in thousandths of a Lorentz unit; the position of the strongest p component (in parentheses) in the same units; the position of the strongest n component in the same units; the calculated values of g for the lower and upper terms.

The notation used for the different types of patterns is as follows:

Type 1: Odd multiplicity, $J_1>J_2$, $g_1<g_2$, shade out.

[9] W. F. Meggers, private communication.

[10] E. Back and A. Landé, *Zeemaneffekt und Multiplettstruktur der Spektrallinien* (Springer, 1925), p. 162.

[11] *M. I. T. Wavelength Tables* (John Wiley & Sons, New York, 1939).

TABLE III.

PARENT TERM	ADDED ELECTRON	TERMS
$4f^2(^3H)$	$6s$	a^4H, a^2H.
	$5d$	a^4K, a^4I, b^4H, a^4G, a^2I, b^2H.
	$6p$	z^4I°, z^4H°, z^4G°, z^2I°, z^2H°, y^2G°.
$4f^2(^3F)$	$6s$	a^4F, a^2F.
	$5d$	c^4H.
	$6p$	y^4G°, z^4F°, z^4D°, z^2G°, z^2F°.
$4f^2(^1G)$	$6s$	a^2G.
	$6p$	y^2H°, z^2G°, y^2F°.
$4f^3$		y^4I°.

Type 2: Odd multiplicity, $J_1>J_2$, $g_1>g_2$, shade in.

Type 3: Odd multiplicity, $J_1=J_2$, $g_1\neq g_2$, symmetrical.

Type 4: Even multiplicity, $J_1>J_2$, $g_1<g_2$, shade out.

Type 5: Even multiplicity, $J_1>J_2$, $g_1>g_2$, shade in.

Type 6: Even multiplicity, $J_1=J_2$, $g_1\neq g_2$, symmetrical.

Type 7a: Limiting cases of types 1 and 2, $J_1>J_2=0$, $g_1\gtrless g_2$, triplet.

Type 7b: Limiting cases of types 1, 2, 3, 4, 5, 6, $g_1=g_2$ triplet. Any multiplicity and all singlet combinations.

In cases where some part of a pattern is missing (usually, because of overlapping from other patterns), the value of g given in parentheses is the average value determined for the appropriate level, and the value of the other g has been calculated from this. All values given in Table I are results from a single set of determinations only.

Table II contains the levels which have thus far been determined, separated into low levels of Group I, middle levels of Group I, low levels of Group II, and middle levels of Group II. The columns, in order, give: the designation of the level in the usual notation, where this has been assigned, and the inner quantum number otherwise; the identification number of the level (in the case of Group II levels); the wave number of the level in reciprocal centimeters; the calculated g value of an ideal level having the quantum numbers given (in the case of Group I levels); the observed final average g value; the probable error in the g value, in the third figure after the decimal, as determined from the agreement of the separate determinations of averages from all the lines arising from that level; the number of lines used in determining the value of g for the level, each usually measured three times. The data of Table II are derived from those of Table I only in part, two other independent sets of determinations having been averaged with these data to get those of Table II.

Not much confidence should be placed in values determined from a single line, on account of the danger of incorrect classification or of misinterpretation of a pattern.

The results given in the present paper do not by any means exhaust the information available from the plates. The interpretation of partially resolved patterns and of mutually perturbing patterns is now proceeding. Levels marked P.B. show extreme Paschen-Back effect.

ENERGY LEVELS

At the present stage of the analysis, the energy system of Ce II consists of two groups of levels which we have not as yet been able to connect. Group I is believed to be the lower of the two by approximately 5000 cm^{-1}. This group consists of levels arising from the electron configurations $4f^26s$, $4f^25d$, $4f^26p$, and $4f^3$. Complete assignments of quantum numbers have been given to all levels, on the basis of Zeeman effect studies, combinations, intervals and intensities. All terms have been assigned to electron configurations and parent terms. In short, the description of these levels by quantum numbers alone seems as complete as is possible by modern spectral theory.

Group II consists of energy levels believed to originate from the electron configurations $4f5d6s$, $4f5d^2$, $4f5d6p$, $4f6s6p$, and possibly also $4f6s^2$, $5d^26s$, and $5d^26p$. Although J values are well

TABLE IV. *g sums for* $4f^2(^3H)6s-{}^4H, {}^2H$.

LEVEL	LS	g VALUES OBS.	LS−Jj	C−O
$a^2H_{4\frac{1}{2}}$	0.909	0.934	0.933	
$a^4H_{4\frac{1}{2}}$	0.970	0.949	0.945	
Sum	1.879	1.883	1.878	−0.004
$a^2H_{5\frac{1}{2}}$	1.091	1.089	1.102	
$a^4H_{5\frac{1}{2}}$	1.132	1.125	1.121	
Sum	2.223	2.214	2.223	0.009

established for these levels, we have given only a few tentative L and S assignments. Electron configuration assignments are made only for the unique cases, such as $^4I^\circ$, which can arise only from $4f5d^2$. Interactions among these levels are so intense that, in general, assignment of additional quantum numbers will have little significance.

The electron configurations and parent terms for the terms of Group I are as given in Table III.

The levels warrant considerable discussion, but for the most part this will be reserved for a later paper. It will serve our purpose to discuss here only those aspects pertinent to the application of the g sum rule.

Tests of the g Sum Rule

The energy separations and intensities associated with a^4H and a^2H indicate that the parent ^3H results from an approximately LS-coupling interaction between the two $4f$ electrons. The $6s$ electron, however, must interact with the parent term with approximately Jj coupling, as its effect (shown by Fig. 1) is to split each ^3H level into two

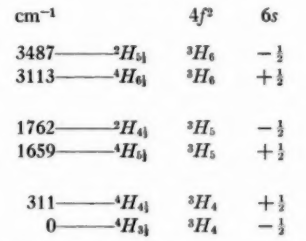

FIG. 1. The terms a^4H and a^2H from $4f^2(^3H)6s$ of Ce II.

levels whose total separation is considerably less than the separation of the levels of the parent term.

TABLE V. g *sums for* $4f^2(^3F)6s-{}^4F$, ^2F *and* $4f^2(^1G)6s^2G$.

LEVEL	LS	OBS.	$C-O$
$a^2F_{2\frac{1}{2}}$	0.857	0.905	
$a^4F_{2\frac{1}{2}}$	1.028	0.986	
Sum	1.885	1.891	−0.006
$a^2F_{3\frac{1}{2}}$	1.142	1.026	
$a^4F_{3\frac{1}{2}}$	1.238	1.213	
$a^2G_{3\frac{1}{2}}$	0.889	1.016	
Sum	3.269	3.255	0.014
$a^4F_{4\frac{1}{2}}$	1.333	1.247	
$a^2G_{4\frac{1}{2}}$	1.111	1.193	
Sum	2.444	2.440	0.004

TABLE VI. g *sums for all low levels of* $J=6\frac{1}{2}$.

LEVEL	LS	g VALUES OBS.	$O-C$
a^4H	1.231	1.225	
a^4K	0.964	0.980	
a^4I	1.107	1.105	
b^4H	1.231	1.210	
a^2I	1.077	1.101	
c^4H	1.231	1.204	
Sum	6.841	6.825	−0.016

If the parent ^3H was a result of ideal coupling, one might then expect the g sum rule to apply to a^4H and a^2H. Owing to the fact that other levels of the same parity and J value are nearby, it is not expected that the experimental sums will check the theoretical sums perfectly. The result of applying the g sum rule to this case, as shown in Table IV, indicates very little perturbation of the levels by other neighboring levels.

For purposes of comparison, the computed values for ideal LS coupling throughout and for $^3H\cdot 6s$ $LS-Jj$ coupling are given. It is seen that, except for the perturbed $a^2H_{5\frac{1}{2}}$ level, the experimental values lie between the computed values of the two ideal coupling schemes, but closer to the $LS-Jj$ case, just as the energy separations indicate they should.

The same comparisons have been carried through for terms from other parents. As is indicated in Table V, the $a^2F_{2\frac{1}{2}}$, and $a^4F_{2\frac{1}{2}}$ levels show little evidence of external perturbation, but the $3\frac{1}{2}$ and $4\frac{1}{2}$ levels show evidence of a large perturbation. However, when the a^2G levels are included in the sums the agreement is fairly good, although still outside the experimental error. This is conclusive evidence of a fairly strong perturbation of the parent $^3_1F_4$ level by the parent $^1G_4$ level. It is also to be noted that if the $4\frac{1}{2}$ levels of a^2H and a^4H are included, the sums agree within the experimental error. This shows that only a very slight perturbation exists between the two groups.

The energy values for the terms resulting from the addition of the $5d$ electron to the parent $4f^2(^3H)$ indicate a much closer approximation to LS coupling throughout than was the case for the addition of the $6s$ electron. This is verified by the g values which, in most cases, are quite close to the ideal LS coupling values. For the J value $6\frac{1}{2}$

TABLE VII. *g sums for* $4f^2(^3H)\cdot 4p - {}^4I^\circ$, ${}^4H^\circ$, ${}^4G^\circ$, ${}^2I^\circ$, ${}^2H^\circ$, ${}^2G^\circ$.

LEVEL	*LS*	*g* VALUES OBS.	*O−C*
$y^2G^\circ_{3\frac{1}{2}}$	0.889	0.975	
$z^4G^\circ_{3\frac{1}{2}}$	0.986	0.900	
$z^4H^\circ_{3\frac{1}{2}}$	0.667	0.712	
Sum	2.542	2.587	−0.045
$y^2G^\circ_{4\frac{1}{2}}$	1.111	1.104	
$z^2H^\circ_{4\frac{1}{2}}$	0.909	0.933	
$z^4G^\circ_{4\frac{1}{2}}$	1.172	1.112	
$z^4H^\circ_{4\frac{1}{2}}$	0.970	0.941	
$z^4I^\circ_{4\frac{1}{2}}$	0.728	0.799	
Sum	4.890	4.889	0.001
$z^2H^\circ_{5\frac{1}{2}}$	1.091	1.107	
$z^2I^\circ_{5\frac{1}{2}}$	0.924	1.019	
$z^4G^\circ_{5\frac{1}{2}}$	1.272	1.210	
$z^4H^\circ_{5\frac{1}{2}}$	1.132	1.036	
$z^4I^\circ_{5\frac{1}{2}}$	0.965	1.005	
Sum	5.384	5.377	−0.007
$z^2I^\circ_{6\frac{1}{2}}$	1.077	1.110	
$z^4H^\circ_{6\frac{1}{2}}$	1.231	1.158	
$z^4I^\circ_{6\frac{1}{2}}$	1.107	1.130	
Sum	3.415	3.398	0.017

all of the theoretical levels except ${}^2K_{6\frac{1}{2}}$ have been found. The *g* sums for the six levels found are given in Table VI.

The *g* sums for $4f^2(^3H)6p$ are shown in Table VII. The agreement is excellent for the $4\frac{1}{2}$ and $5\frac{1}{2}$ levels, where the sum is for five levels in each case. The sum for *J* values of $3\frac{1}{2}$ shows the effect of a small perturbation, probably from the levels of $4f^2(^3F)6p$ which are close by.

The *g* sums for the $4f^2(^3F)6p$ levels cannot be tested, as all of the levels have not yet been discovered. It is apparent, however, by inspection of *g* values, intervals and intensities, that the perturbations of the levels of this group are much larger than for those based on $4f^2(^3H)$.

ACKNOWLEDGMENT

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Ninth Summer Conference on Spectroscopy and Its Applications

A NINTH Summer Conference on Spectroscopy and Its Applications will be held at the Massachusetts Institute of Technology on July 21, 22, and 23 of this year. Morning and afternoon sessions will be held during three days, with discussions by qualified experts on spectroscopic analysis of materials and on other applications of spectroscopy to biology, medicine, chemistry, metallurgy, mineralogy, and to industrial and engineering problems.

This conference will repeat to a considerable extent the type of program presented at the conferences held during the past eight summers. The early sessions will be devoted principally to applications of spectroscopy to the analysis of materials. Other topics to be considered are absorption spectrophotometry, photographic photometry, biological and chemical effects of spectral radiation, industrial applications of spectroscopy, analysis of spectra and the measurement of wave-lengths. The more technical and theoretical papers will be reserved for the later sessions.

As soon as it has been prepared, copies of the detailed program of the conference will be sent to anyone interested. Attendance at the conference will be limited to 200, and experience in previous years indicates that those who plan to attend from a distance should signify their intention in advance, to insure availability of space. No fees are charged for attendance.

Requests for copies of the program, for reservations, or for further information, should be addressed to Professor G. R. Harrison, Massachusetts Institute of Technology, Cambridge, Massachusetts.

Annual Report of the Secretary of the Optical Society of America, Inc. for the Calendar Year 1940

Membership Record

January 1, 1940 to January 1, 1941

	Honorary	Regular Annual Payment	Regular Life Payment	Associate Individual Annual Payment	Associate Corporation	Total
Membership as of record in the books of the Secretary 1–1–40	2	419	6	213	10	650
Gains during 1940						
By election as of 1–1–40	0	0	0	64	0	64
By election as of 1–1–41	0	0	0	199	0	199
By reinstatement	0	0	0	1	0	1
By transfer 10–2–40 as of 1–1–41	0	14	0	0	0	14
By transfer 10–2–40 as of 1–1–41	2	0	0	0	0	2
Losses during 1940						
By death	0	6–	0	1–	0	7–
By resignation as of 1–1–40	0	8–	0	6–	0	14–
By resignation as of 12–31–40	0	6–	0	5–	0	11–
By transfer 10–2–40 as of 1–1–41	0	0	0	14–	0	14–
By transfer 10–2–40 as of 1–1–41	0	2–	0	0	0	2–
By separation for non-payment as of 1–1–40	0	5–	0	6–	0	11–
Membership as of record 1–1–41	4	406	6	445	10	871

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Jelley, Edwin E. (1230 A)
Karwoski, T. F. (1217 A)
Nelson, C. N. (1231 A)
Park, Robert H. (1219 A)
Richardson, Earle E. (1232 A)
Shillaber, Charles P. (1244 A)
Spence, John (1188 A)
Van den Akker, J. A. (1242 A)
Wald, George (1208 A)
Wallace, S. Rains (1226 A)

Elected to Honorary Membership by Action of the Board of Directors taken by Letter Ballot of November 2, 1940, effective as of January 1, 1941

Lyman, Theodore (161 R)
Raman, C. V. (836 R)

Deaths During 1940

Dr. Calvin B. Coulter (1085 R) on May 10
Dr. Frederick E. Fowle (646 R) on April 22
Mr. L. J. R. Holst (102 R) on April 12
Professor Clement Moran (639 A) on January 26
Professor A. deF. Palmer (728 R) on January 13
Professor T. R. Wilkins (529 R) on December 10
Professor C. W. Woodworth (164 R) on November 19

Resignations during 1940 as of January 1, 1940

G. R. Alden (1072 A)
S. H. Gage (710 R)
Percy Hodge (475 R)
James E. Ives (316 R)
F. W. Lutz (674 R)
Dana P. Mitchell (684 R)
G. A. Mitchell (901 A)
R. A. Porter (310 R)
C. H. Ross (1020 A)
A. B. Rotenberg (1153 A)
Arthur C. Ruark (327 R)
C. H. Stone (735 A)
T. Smith Taylor (519 R)
R. W. Vose (1019 A)

Resignations during 1940 as of December 31, 1940

E. V. Collins (1293 A)
John N. Dalton (1298 A)
H. G. Gale (143 R)
H. M. Johnson (66 R)
J. F. Meyer (596 R)
Vincent H. Morris (1333 A)
C. A. Skinner (122 R)
Wilmer Souder (394 R)
F. A. Tompkins (881 A)
F. P. Walters (1117 A)
Robert D. Nutting (967 R)

(Signed) ARTHUR C. HARDY, *Secretary*

Annual Report of the Treasurer of the Optical Society of America for the Fiscal Year 1940

CASH TRANSACTIONS JANUARY 1, 1940 TO DECEMBER 31, 1940

Receipts

Dues	$2,905.06	
Dues in advance	2,224.20	
Interest on unallocated securities	920.00	
Interest on bank balance	54.44	
Interest on Ives Medal Fund Bonds	15.00	
Interest on A. Lomb Memorial Fund Bonds	79.99	
Bank interest on Ives Medal Fund	2.01	
Bank interest on A. Lomb Memorial Fund	4.21	
Bank interest on Local Committee Fund	1.73	
Local Committee Fund	193.00	
		$6,399.64

Disbursements

Secretary's expense	$1,094.25	
Secretary's travel expense	133.72	
Treasurer's expense	269.07	
President's expense	27.33	
Journal publication expense	3,914.65	
Journal editorial expense	1,401.63	
Membership Committee expense	46.45	
Ives Medal Fund expense	—	
A. Lomb Memorial Fund expense	18.50	
Local Committee expense	90.05	
Program Committee expense	—	
Specially Appropriated Expenditures		
Treasurer's deficiency appropriation	602.36	
Inter-Society Color Council dues	25.00	
A.S.A. Committee Z38 on Photography	100.00	
Moving editor's office—Ithaca to Cambridge	15.75	
Dies for A. Lomb Medal	500.00	
		$8,238.76
Excess of disbursements over receipts		$1,839.12

Balances—December 31, 1939		
Cash in bank	$6,472.62	
Ives Medal Fund interest	95.92	
A. Lomb Memorial Fund interest	162.21	
Local Committee Fund	78.13	
		$6,808.88
Cash Balance, December 31, 1940		$4,969.76

Ives Medal Fund

Income

Principal sum	$500.00	
Cash interest balance, December 31, 1939	95.92	
Bond interest accrued during 1940	15.00	
Bank interest on cash balance, 1940	2.01	
		$ 612.93
Disbursements		
.....		.....
Balance, December 31, 1940		$ 612.93

Note: The principal sum of $500.00 is invested in U. S. Treasury Certificates. They are included in the bonds reported in the accompanyin Investment Statement.

Adolph Lomb Memorial Fund

Income

Invested principal, including taxes, fees, etc. in Dominion of Canada 3¼% bonds, January 1, 1940.	$1,919.06
Uninvested principal, bond and bank interest January 1, 1940	162.21
Bond interest accrued during 1940	79.99
Bank interest accrued during 1940	4.21
	$2,165.47
Disbursements	
Net loss, including commissions, taxes, etc. on sale of Dominion of Canada 3¼% bonds, October 8, 1940	$ 225.24
Cost of Medal awarded David MacAdam, November 18	18.50
	$ 243.74
Balance, December 31, 1940	$1,921.73

Note: The cash proceeds of the sale of the Dominion of Canada Bonds amounting to $1693.82, are in the hands of our Trust Officer awaiting suitable investment.

Local Committee Fund

Income

Cash balance, December 31, 1939	$ 78.13	
Collections at Rochester meeting	193.00	
Bank interest accrued during 1940	1.73	
		$ 272.86
Disbursements		
Expenses for Fall Meeting at Rochester		90.05
Balance, December 31, 1940		$ 182.81

Securities Held by the Security Trust Company of Rochester as Agent for the Optical Society of America, Inc.

	Cost	Market Value 2-3-41	Dividends During 1940	Rate of Return Against Cost
Bonds				
$7000 Atchison, Topeka & Santa Fe 4%	$ 6,916.25	$ 7,630.00	$280.00	4.05%
$5600 U. S. Treasury Certificates 3¼%	5,625.67	6,020.00	182.00	3.23%
$8000 U. S. Treasury Certificates 3%	8,261.30	8,872.00	240.00	2.91%
Total Bonds	$20,803.22	$22,522.00	$702.00	3.37%
Stocks				
10 shares Eastman Kodak Common	$ 1,731.40	$ 1,305.00	$ 60.00	3.47%
40 shares General Electric Common	1,871.35	1,320.00	68.00	3.98%
35 shares Kennecott Copper Common	1,729.53	1,120.00	105.00	6.06%
Total Stocks	$ 5,332.28	$ 3,745.00	$233.00	4.36%
Total Stocks and Bonds together	$26,135.50	$26,267.00	$935.00	3.73%

February 15, 1941

The undersigned, having been duly appointed to audit the books of the Treasurer of the Optical Society of America, Inc., from January 1, 1940 to December 31, 1940, hereby certify that the Treasurer's report of receipts, disbursements, and investments is correct as shown by records, vouchers, bank statements, etc.

(Signed) Carl L. Bausch, C. W. Frederick, *Auditors*

Report of the Editor of Publications

Statement of Operations of the Journal of the Optical Society of America for the Year Ending December 31, 1940

A. In Account with American Institute of Physics, Incorporated

Income			
Subscriptions		$4,468.12	
Net Income from Published Articles, Schedule 1		1,085.61	
Net Income from Sale of Back Numbers		1,023.97	
Total Income			$ 6,577.70
Expense			
Printing and Mailing		$7,123.77	
Engraving		1,060.25	
Editorial Mechanics:			
Salaries	$772.81		
Office Expense	220.97	993.78	
Subscription Expense:			
Salaries	$655.06		
Office Expense	196.26	851.32	
Other Expenses:			
Auditing and Insurance	$ 66.22		
Inventory and Miscellaneous	83.96	150.18	
Total Expense			$10,179.30
Excess of expense over income			$ 3,601.60
Service charge of American Institute of Physics (15%)			1,721.54
Contribution to A. I. P. in addition to service charge (5%)			573.84
Net Cost of Editorial Mechanics, Printing and Distributing Journal			$ 5,896.98

B. Expenses in editorial office (Cambridge)

Salaries	$1,000.00	
Operating Expenses (App. B)	317.00	
Total in Cambridge Office		1,317.00
Total net cost of journal for 1940		$ 7,213.98
Total cost of journal for 1940 for 667 pp. of editorial material, or $20.70 per page		$13,791.68

Number of pages published	*1940*	*1939*	*1938*	*1937*
Text and index	666	550	508	437
Covers and front matter	72	72	56	72
Total	738	622	564	509

Cumulative Index of the Journal of the Optical Society of America

Income			
Sales of Cumulative Index			$ 15.70
Expense			
Handling and Mailing	$ 5.54		
20% Service charge	1.11		6.65
Net Income			$ 9.05

SCHEDULE I

Net Income from Published Articles

Income		
Publication Charges	$ 916.20	
Sale of Reprints	1,280.89	
Total Income		$ 2,197.09
Expense		
Printing and Mailing Costs of Reprints	$ 989.05	
Salaries	95.22	
Office Expense	27.21	
Total Expense		1,111.48
Net Income from published articles		$ 1,085.61

Net Income from Sale of Back Numbers

Income		
Sale of Back Numbers		$ 1,210.12
Expense		
Salaries	$ 140.78	
Office Expense	45.37	
Total Expense		186.15
Net Income from sale of back numbers		$ 1,023.97

(Signed) GEORGE R. HARRISON, *Editor*

Proceedings of the Cambridge Meeting of the Optical Society of America

FEBRUARY 21–22, 1941

JOINT MEETING WITH THE AMERICAN PHYSICAL SOCIETY

A JOINT MEETING of the Optical Society of America and the American Physical Society was held in Cambridge, Massachusetts, on February 21–22. The meeting was opened with a Symposium of invited papers relating to color, over which President Gibson presided. The titles of the invited papers were as follows:

1. **Spectrophotometry, a Basis for Color Measurement.** BY ARTHUR C. HARDY, *Massachusetts Institute of Technology.*
2. **Colorimetry and the Standard Observer.** BY DAVID L. MACADAM, *Eastman Kodak Company.*
3. **Color Perception and Its Relation to Color Measurement.** BY DEANE B. JUDD, *National Bureau of Standards.*

In spite of evident absences due to defense activities, there was an audience of approximately 400 at the opening session. These invited papers, which took the form of experimental lectures illustrated by demonstrations, lantern slides, and motion pictures, are abstracted by the Chairman of the Committee on Meetings and Programs in the following words.

Professor Hardy's paper on "Spectrophotometry, a Basis for Color Measurement," called attention to the introductory character of his own remarks and the relatively greater importance of the papers of the subsequent speakers. He made it clear that, under the definition of the term "color" which has achieved the widest acceptance, a spectrophotometer is incapable of measuring color. That it constitutes a basis for the measurement of color was to be shown more clearly in the second paper by Dr. MacAdam, and consequently Professor Hardy restricted his remarks to the art of spectrophotometry itself.

The fact that colored objects reflect light differently at different wave-lengths was illustrated by experiments, and the qualitative relationship of a spectrophotometric curve to the color of objects was demonstrated. The very valuable application of science in preserving for future generations, unchanged and unpolluted, the sacred colors of a well-known technical school was cited as an outstanding instance of the benefits of spectrophotometry to mankind, appealing to humanity, not on a materialistic, but on a high idealistic level.

A characteristic industrial color problem is the control of a process of production so that different lots or shipments are identical in their color characteristics, and Professor Hardy showed how the spectrophotometer is admirably suited to the solution of this problem, even in the case of turbid media such as moulded plastics. It was brought out that the industrial use of spectrophotometry in this country is of the order of a million curves per year, the rate of increase appearing to be exponential.

The automatic spectrophotometer developed in Professor Hardy's laboratory was shown in colored motion pictures which vividly illustrated the speed and ease with which these important fundamental data can be obtained for either transparent or opaque materials. Professor Hardy emphasized the point that, under comparable conditions, if materials have identical spectrophotometric curves, they will be identical in color, but that the converse of this is not necessarily true. He showed two bolts of dyed cloth which had conspicuously different spectrophotometric curves, but under the illumination of the lecture room appeared to be excellent color matches. The color difference between them was, however, clearly brought out when they were shown under artificial daylight. Finally, Professor Hardy showed a motion picture of a mechanical integrator which, operating in conjunction with the spectrophotometer, is capable of providing automatically the numerical values which are used in the tristimulus specification of color, discussed by the next speaker.

The paper on "Colorimetry and the Standard Observer," given by Dr. David L. MacAdam of the Eastman Kodak Company, reemphasized the point which Professor Hardy had made, that spectrophotometric measurements are purely physical in character, as are also radiometric

measurements of the light incident on colored objects. The standard illuminants which have been chosen for convenience by the International Commission on Illumination were described. In order to convert these physical data into values related to color, the reactions of the human observer to radiant energy under definite experimental conditions must be obtained. The subjective nature of the response of a person to an external stimulus was mentioned by Dr. MacAdam in discussing briefly the difficulties and limitations of a purely introspective approach to color, particularly the impossibility of placing such characterizations on a numerical basis. The useful artifice of the Standard Observer has therefore been introduced: that is, an observer whose responses are quantitatively defined by functions based on the experimental examination of a large number of "normal" observers. The point of view which Dr. MacAdam presents is that light is the aspect of radiant energy of which human observers are aware through visual sensations, and that color consists of the characteristics of light other than its spatial and temporal inhomogeneities. Color is characterized by three quantities: a photometric quantity, dominant wave-length, and purity.

The possibility of providing a wide range of color stimuli by mixing two and then three colored lights was illustrated, and the difference between this "additive" mixture and the "subtractive" mixture obtained by mixing dyes or pigments was emphasized. The specification of color is based on the additive mixture of three standard stimuli, and the forms of expressing the results according to the conventions of the International Commission on Illumination (1931) were shown.

The tristimulus specification of color involves the obtaining of three integrals, and the various and commonly-used ways of obtaining these integrals was illustrated. Finally, Dr. MacAdam showed that this method of obtaining and treating data quite adequately and quantitatively accounted for the experiment with the cloths of different spectrophotometric curves which did match in color under one illuminant but did not match under another.

Dr. MacAdam pointed out that the arts of color photography and color reproduction are based on the fact that a color match can be established without requiring a spectrophotometric match. He closed his talk by showing a motion picture of color measurements in the Eastman Kodak Laboratories and their direct application to the study of color reproduction in color prints.

The third paper of the symposium, "Color Perception and its Relation to Color Measurement," was presented by Dr. Deane B. Judd of the National Bureau of Standards who took as his text "Even though you know the color, you still don't know what you'll see." Dr. Judd did not attempt to refute the statements of the previous speakers, which indeed he endorsed explicitly by emphasizing the very great usefulness of colorimetry in its present state, both to science and to industry. His paper was directed to showing the limitations of our present knowledge of color as it is perceived. Dr. Judd began by describing how, on a purely psychological basis, it is possible for an observer to arrange a series of colored objects in definite orders, and in particular to arrange surface colors in a three-dimensional space in which there would be variations in hue, in saturation, and in lightness. He then demonstrated by experiments that a specification of dominant wavelength, purity, and luminous reflectance would not suffice to define the psychological attributes hue, saturation, and lightness.

The next experiment illustrated the efforts of Jack and Jill to reach a compromise on the use of oleomargarine. Dr. Judd showed first a yellow card representing butter, and a white card representing oleomargarine. When the samples were placed on a black background and illuminated by yellow light, both were perceived as yellow. On the other hand, both samples were perceived as white when placed on a white background and viewed under the yellow light. It was concluded that Jack could not make the oleomargarine look yellow like butter by using a yellow light in the dining room unless Jill would permit the use of a black tablecloth and black dishes.

The extremely important effect of the background on the appearance of colors under chromatic illumination was further illustrated by color cards viewed first with a white background

and then a black one. In nearly all cases, observers were aware of a considerable change in hue which, however, required an appreciable length of time to develop. These experiments show that modern colorimetry does not take into account the effect of surrounding colors, but experimental work intended to elucidate this relationship is in progress.

Even if this factor of background is taken into account, it is sometimes necessary to consider what the observer thinks he sees. Priest's early reference to gray skies and white snow and to blue skies and white snow were recalled. The phenomenon of "Skinner's Lake" was mentioned, a view in which a portion of the landscape was seen either as "a limpid pool of shimmering blue," or "a bank of snow."

Summing up, Dr. Judd repeated that "Even if you know the color, you don't know what you will see. Colorimetry has not yet learned to take into account what the observer is thinking. You can do a lot with a spectrophotometer, but this does not include mind-reading."

Joint Dinner

The dinner of the combined Societies at the Hotel Commander was attended by about two hundred members of the Societies and their guests and, "as is the custom of the Physical Society," there were no after-dinner speakers. Nevertheless, there were "remarks" by those whom circumstance or duress had led to sit at the head table. Dean Pegram, President of the American Physical Society, presided and referred briefly but effectively to the qualifications of the various men so honored. Remarks of welcome to the visiting societies were made by Professor Van Vleck and by Professor Slater of the Local Committee on behalf of Harvard University and the Massachusetts Institute of Technology.

The activities of the National Defense Research Committee were described by President Karl T. Compton, of the Massachusetts Institute of Technology, who indicated the general character of the organization and outlined its methods of operation. The daily trials and tribulations, and perhaps triumphs, of a Section leader of this organization were graphically portrayed by Professor G. R. Harrison, in charge of the Section on Instruments.

Contributed Papers

The abstracts of the contributed papers are printed elsewhere in this issue and it is not intended that this review should give them special mention. On the other hand, it seems appropriate to refer briefly to the Report of the Colorimetry Committee of the Optical Society, which was presented by its chairman, Dr. L. A. Jones, on Saturday morning. This report, which presented to the members of the Society the present status of the work of the Committee, included a brief historical survey of the work of the Committee and of the evolution of the report, the writing of which is the Committee's chief *raison d'être*. An attempt is being made to write a report which will be more comprehensive and less strictly professional than the earlier O.S.A. Colorimetry Report. After much discussion, it has seemed necessary to modify the definition of color adopted in the earlier report. The reasons for abandoning the psychological definition in favor of a psycho-physical definition were indicated by Dr. Jones, who stressed the fact that this was not a compromise measure but an expression of what had seemed to the Committee after long and careful deliberation to be the most logical basis on which the report can be written. There followed a description of the contents of the proposed report and an indication of the present status of the different parts. In general, it appears that the viewpoint from which the report is to be written has been determined; the general outline has been prepared; and large portions of the report (including tables of numerical data) have been completed. Some portions are still unwritten and others require further revision. It appeared, however, that the major part of the work of the Committee had been accomplished, and that the completion of the report may be expected in the not-too-distant future.

Meeting of the Board of Directors

A meeting of the Board of Directors was held on Thursday, February 20. The Minutes of this meeting are appended.

Arthur C. Hardy, *Secretary*

Minutes of the Seventeenth Meeting of the Directors of the Optical Society of America, Incorporated

CAMBRIDGE, MASSACHUSETTS, FEBRUARY 20, 1941

IN addition to the transaction of routine business and the receiving of reports of committees and representatives, the following items of general interest to members of the Society were taken up.

Announcements by Secretary

Results of letter ballots:

1. Appointment of

R. C. GIBBS, *Chairman*	H. P. GAGE,
I. H. GODLOVE,	W. C. GRANVILLE,
D. B. JUDD,	A. C. HARDY,
BLANCHE BELLAMY,	L. A. JONES,
R. E. ESTEY,	DOROTHY NICKERSON.

as representatives on the Inter-Society Color Council, each for a term to expire in October, 1942, with R. C. Gibbs, I. H. Godlove, and D. B. Judd serving as voting delegates.

2. Appointment of the following Associate Editors of the Journal for the indicated terms,

Term ending December 31, 1941

THOMAS A. WRIGHT, R. BOWLING BARNES.

Term ending December 31, 1943

BURT H. CARROLL, M. G. MELLON.

Term ending December 31, 1945

WALLACE R. BRODE, RALPH A. SAWYER.

3. Appointment of A. G. Worthing as Chairman of the Glossary Committee.

4. Appointment of C. W. Gartlein as a member of the Membership Committee for a term to expire in October, 1941.

5. Election to Honorary Membership in the Optical Society of America of:

PROFESSOR THEODORE LYMAN, SIR C. V. RAMAN.

Announcement of Appointments by President

1. Nominating Committee for Election of Officers for terms beginning October, 1941

R. C. GIBBS, *Chairman*

W. E. FORSYTHE,	L. V. FOSTER,
C. E. FOSS,	H. T. WENSEL.

2. Representative on Division of Physical Sciences of National Research Council for a term to expire on June 30, 1944

R. C. GIBBS.

Membership

The Secretary reported informally that between January 1, 1941 and February 20, 1941, 47 new members had been elected to associate membership and one former member had been re-instated. ARTHUR C. HARDY, *Secretary*

Abstracts of Contributed Papers Presented Before Sessions of the Optical Society of America

1. An Experimental Study of the Rate of a Moving Atomic Clock. II. HERBERT E. IVES AND G. R. STILWELL, *Bell Telephone Laboratories.*

A continuation of the work on the Doppler effect in high speed hydrogen canal rays previously described.[1] The voltage range has been extended from the previous 20,000 to 43,000 by the use of multiple-gap electrodes, whereby the frequency shift has been approximately doubled, to over 0.1A. The previously found relationship $\nu' = \nu(1-v^2/c^2)^{\frac{1}{2}}$ is confirmed over this whole range. In addition to the H_2 and H_3 particles previously studied the line due to H_2O, of mass 19, has been observed, showing the same frequency shift with velocity. A study has also been made of the shift of the central, "undisplaced" line due to velocity imparted by impact of the canal rays. A detectable shift is shown which becomes negligible when the mirror used to reflect the receding rays is in place. Observations were also made with the discharge receding from the observer, and thus seen approaching in the mirror, reversing the previous procedure. These observations agree with the previous series and show the result to be unaffected by the particular optical arrangement used.

[1] J. Opt. Soc. Am. **28**, 215–226 (1938).

2. Electro-Optical Field Mapping. HANS MUELLER, *Massachusetts Institute of Technology.*

Recent investigations[1] have revealed that certain colloidal solutions of bentonite possess an extremely large electro-optical effect with a Kerr constant which is between 10^6 to 10^7 times larger than that of nitrobenzene. Though the electric birefringence approaches a saturation value for fields above 100 volts/cm it is easy to create phase shifts between the ordinary and extraordinary ray amounting to several multiples of 2π. This fact makes it possible to investigate the field distribution between electrodes

immersed in the colloid by a simple optical method analogous to photoelasticity. Observation through crossed Nicols of a plane inhomogeneous electric field shows two orthogonal systems of lines. The black isoclinic lines are the loci of points where the field direction is in the plane of polarization of either the polarizer or the analyzer. The colored isochromatic lines determine the electric field intensity in any point. Since the colloidal particles are charged, it is necessary to apply a.c. voltages across the electrodes. Nevertheless sharp diffraction lines can be obtained either by using an a.c.-operated light source or by using a field frequency above 1000 cycles per sec. for which the colloidal particles are too sluggish to follow the alternations of the field.

[1] H. Mueller, Phys. Rev. **55**, 508, 792 (1939). F. J. Norton, Phys. Rev. **55**, 668 (1939). H. Mueller and B. W. Sakmann, Phys. Rev. **56**, 615 (1939).

3. Analysis of Optical Systems. SEYMOUR ROSIN AND ORRIN H. CLARK, *Columbia University.*

The cardinal points of an optical system containing an arbitrary number of coaxial components are obtained from a general formula for the lateral magnification by assigning the appropriate values to the latter. The magnification formula is expressed in terms of a characteristic determinant which contains the focal lengths and separations of the components in a simple manner. The analysis of optical systems in which it is desired to introduce fractional changes in the values of the optical constants is performed by expanding the expressions for the cardinal points in a Taylor's series about the original values. In particular, the formulae give specific and useful expressions from which may be calculated the effect upon the focal length and the positions of the focal points of the combined system, when one or more of the radii, or of the internal separations, or of the indices of refraction are varied.

4. Illumination in the Focal Plane. FRANK BENFORD, *Research Laboratory, General Electric Company.*

The rated speed of a lens tells only the relative illumination at the center of the film, and the illumination at points removed from the optic axis is often greatly influenced by factors other than the f: number. If the lens assembly has a length that is comparable with its diameter the end mounting rings play an important part in reducing the off-axis illumination by occulting the diaphragm opening and frequently the end rings rather than the diaphragm play the dominant part in limiting the light reaching the edge of the film. This paper is a report on angular tests made on a group of 36 lenses to determine their angular illumination characteristics, for it has been found that every lens examined departs from the $\cos^4\theta$ relation. A second characteristic studied is the departure from rated f: number when the image is at other than the principal focal point. In this part of the work the lens is assumed to be corrected for coma, that is, the principal surface on the image side of the lens is a sphere centered on the principal focal point. The same rule applies to the new Schmidt camera which has so greatly extended the boundaries of practical high speed lens design.

5. The Use of Modulated Lamps in Photometry. S. Q. DUNTLEY, *Massachusetts Institute of Technology.*

To a useful first approximation, the radiant flux emitted by a gaseous conduction lamp is proportional to the current passing through it. If such a lamp is supplied with an unvarying unidirectional current to which is added a cyclically varying current, the radiant output will consist of a constant glow with a superimposed cyclically varying flux, the wave form of which corresponds to that of the cyclic current if the total current through the lamp is never reduced below a limiting minimum value. Two such lamps are connected so that they share direct current from the same battery but have impressed on them alternating currents of opposite phase. Equal amounts of light from both lamps enter a radiation mixing enclosure, the wall of which is viewed by a photoelectric cell. When the modulations are of equal amplitude, a flickerless condition exists at the photo-cell. If an absorber is inserted in the path of one lamp, a flicker results. By reducing the modulation of the unobstructed lamp, by increasing the modulation of the obstructed lamp, or by both, a flickerless condition may be restored. This change in modulation is conveniently accomplished by means of a transformer having a sliding contact on its secondary, the position of the slider serving as an index of the photometric reading. An extension of the above principle to a system of four lamps permits the use of a method described by Stockbarger[1] to isolate any spectral region emitted by the lamps without the use of a band-pass filter or a monochromator.

[1] D. C. Stockbarger, Rev. Sci. Inst. **1**, 76 (1930).

6. A Simple Method for Investigating the Optical Modulation of a Gaseous Conduction Lamp. D. B. CAMERON AND S. Q. DUNTLEY, *Massachusetts Institute of Technology.*

It was stated in the previous paper that, to a useful approximation, the wave form of the modulated flux from a gaseous conduction lamp corresponds to that of the cyclic component of the current. If the presence or absence of flicker is determined by means of a balance motor in the manner described by Hardy,[1] the introduction of harmonics by the lamp will not affect photometric measurements obtained in the manner there described. To study the effective optical modulation of a gaseous conduction lamp when used under these conditions, the photometer described by Hardy[2] is used to generate an accurately known modulated flux in its integrating sphere. Light from the gaseous conduction lamp, modulated with opposite phase, is allowed to enter the sphere. After equalizing the total quantity of the two fluxes, the photometer balances in the normal manner automatically. The effective optical modulation (m) is related to the transmission scale reading of the photometer (T) by the equation $m=(1-T)/(1+T)$. The electrical modulation imposed on the lamp is measured directly. By using this method, curves of electrical modulation *vs.* effective optical modulation have been obtained for several types of lamps suitable for photometric purposes.

[1] Hardy, J. Opt. Soc. Am. and Rev. Sci. Inst. **18**, 96 (1929).
[2] Hardy, J. Opt. Soc. Am. **18**, 305, 1935).

7. A Modulated Lamp Densitometer. S. Q. DUNTLEY AND JOHN E. TYLER, *Massachusetts Institute of Technology.*

An automatic direct-reading photoelectric densitometer capable of both reflectance and transmittance measurements has been constructed using the modulated lamp principle described in the foregoing papers. This instrument contains an eight-inch diameter integrating sphere with a window at which the sample is presented. For reflectance measurements, a beam of light from a small neon glow lamp is directed normally on the surface of the sample. Reflected light is collected by the sphere and is mixed with light from a second neon tube. The photo-cell amplifier and balancing system described by Hardy[1] is used to control a Variac autotransformer, the dial of which is marked in density units. When transmission measurements are desired, an argon glow lamp carried by a hinged arm is lowered over the sample, which is placed at the same window in the integrating sphere as is used for reflectance measurements. Light transmitted by the sample is mixed with flux from a second argon lamp. An argon lamp is used for transmission measurements instead of a Neon lamp because it emits more copiously in the spectral region where positive photographic materials are most sensitive. The measured area of the negative may be varied from one to 1/100 square inch.

[1] Hardy, J. Opt. Soc. Am. and Rev. Sci. Inst. **18**, 96 (1929).

8. Report of the Optical Society of America Committee on Colorimetry. L. A. JONES, *Chairman.*

A short review of the Committee's activities will be given together with a summary of the present status of the report. The structure of the report as planned will be outlined and a brief summary of the subject matter of each of the chapters which are now nearing completion will be given. In the preface of the report, the chairman plans to discuss at some length the historical background and the evolution of the report. In the first chapter an elementary discussion of the whole field is given and in the second the same ground is covered in a more detailed fashion. These chapters are designed specifically to introduce the reader gradually to the more advanced treatments and thus to make the report of interest to readers not specialized in the field of colorimetrics. The third chapter is devoted to a discussion of the eye and its associated nerve systems. The fourth chapter is devoted to a rather detailed discussion of the psychological aspects of the subject, while the fifth treats the physics of color, particularly the measurement and specification of physical stimuli. In the sixth chapter the psychophysical viewpoint is presented. This chapter contains the psychophysical definitions of light and color as formulated by the Committee, and treats rather exhaustively the consequences of assuming the psychophysical point of view, which constitutes the unique feature of this report as compared with the previous one published in 1922. In the seventh chapter it is planned to include all the numerical data, physical, psychological, and psychophysical, necessary for a complete and comprehensive treatment of the subject of colorimetry.

9. Rods and Cones in Visual Threshold and Adaptation. GEORGE WALD,* *Biological Laboratories, Harvard University.*

Any two colored lights are effective in some constant energy ratio for stimulating cones, and in some other fixed ratio for stimulating rods, due to the different spectral sensitivities of these receptors. Such ratios can be used to analyze precisely the participation of rods and cones in visual functions. We have measured the absolute thresholds for light sensation, regardless of color, alternately through a yellow (Wratten 73) and a violet (Corning 511) filter. A 10-volt, 7.5-ampere photo-cell exciter lamp behind a Jena BG 9 heat filter served as source. At 12.5° from the fixation point (0.75° field) the cone threshold for violet lies about 0.5 log unit above that for yellow, the rod threshold for violet about 0.8 log unit below that for yellow. As the field is moved centrally these separations remain unchanged until the border of the macula is reached at about 2° excentrically. Inside this limit the cone separation widens to an eventual 1.2 log units within the fovea, and the rod separation correspondingly narrows, due to disproportionate absorption of the violet light by macular pigment. Perimetric measurements in the completely dark adapted eye show that beyond about 2° from the fixation point rods govern the threshold completely. Inside the 2° radius cones participate in the threshold increasingly, and finally exclusively as the fovea is entered. The dark adaptation of a series of retinal areas has been measured by these methods. Within the fovea a parallel pair of curves is obtained, pure cone in separation throughout. In peripheral fields, after a period of pure cone separation the curves approach each other, cross, then re-widen to the pure rod separation. The region of mixed rod and cone function is extensive; at 0.4°–2° excentrically it includes the major portion of dark adaptation. The presence or absence of color sensation is not a reliable criterion of cone or rod function. Color sensations are still experienced when the shift from cone to rod vision is well advanced, and cease long before the shift has been completed.

* This research has been supported in part by the Josiah Macy, Jr., Foundation.

10. Solid Geometry in Color Space. ISAY A. BALINKIN, *Physics Department, University of Cincinnati.*

The possibility of specifying uniquely any surface color by its three attributes or parameters leads to a three-dimensional color space where each point represents a color. In the Munsell system the parameters are designated by Hue (H), Value (V), and Chroma (C). Any color, K, can be expressed, therefore, as $K=f(H, V, C)$ or HVC. A change of $\pm\Delta H$, $\pm\Delta V$, and $\pm\Delta C$ in any one, two or three parameters will give a "color-lattice" consisting of 26 points such as H_1VC, H_2VC, HV_1C, HV_2C, HVC_1, HVC_2, and so on, where $H_1=H+\Delta H$, $H_2=H-\Delta H$, $V_1=V+\Delta V$, $V_2=V-\Delta V$, $C_1=C+\Delta C$, $C_2=C-\Delta C$. Such a lattice will not represent, however, a system of colors visually equidistant from the original color HVC, even when $\Delta H=\Delta V=\Delta C$. Considering the geometrical properties of a homogeneous and isotropic psychophysical color

space[1] where the distance between any two points is directly proportional to the visual perception of color differences the following propositions can be derived. *Proposition 1.*—A spherical surface is a locus of colors visually equidistant from a certain color located at the center of the sphere. Because of the finite value of least perceptible color differences, the number of points on the sphere is also finite. These points may be considered to form a "color-lattice" with a number of points directly proportional to the square of their distance from the center of the sphere. *Proposition 2.*—The centers of polygons of a regular pentagonal dodecahedron is a lattice of 12 colors which are approximately[2] visually equidistant from the neighboring colors including the color located at the center of the solid. This means that for any given color there is a "color-lattice" of only twelve colors which are visually nearly equidistant. *Proposition 3.*—The vertices of a regular tetrahedron is a lattice of four colors which are visually equidistant from *any* one of the neighboring colors. This means that it is possible to choose a "color-lattice" of only four colors so that the visual color difference between any two of them is exactly the same. In other words if K_1, K_2, K_3 and K_4 is one of these "color-lattices," then

$$K_1-K_2=K_1-K_3=K_1-K_4=K_2-K_3=K_2-K_4=K_3-K_4.$$

If it were possible to show that any five colors could satisfy this requirement, it would be necessary to assign four dimensions to the color space. The considerations of geometrical symmetry in a homogeneous and isotropic psychophysical color space may lead to a rational system of color harmony.

[1] I. A. Balinkin, "Industrial color tolerances," Am. J. Psych. **52**, 428 (1939).
[2] Lord Kelvin, "Baltimore Lectures," 602 (1904).

11. Use of Didymium Glasses for Wave-Length Calibration of Recording Spectrophotometers. KASSON S. GIBSON AND HARRY J. KEEGAN, *National Bureau of Standards.*

On recording spectrophotometers it is desirable to obtain wave-length calibration of the instrument while in operation. This is possible by running the curve for a sample of didymium glass on the same sheet as the curves for the sample being measured, provided the wave-lengths of maximum absorption of certain of the didymium bands are known for the slit widths used. In connection with tests of four Corning No. 512 didymium glasses for this purpose, the following average wave-lengths of maximum absorption have been determined, applicable to 10 millimicron slits on recording spectrophotometers: 441.6, 475, 528.7, 585.4, 684.8, and 745 millimicrons. The uncertainty in these values is of the order of 1 millimicron. While these values cannot be guaranteed valid for all Corning 512 glasses, the agreement obtained with the four samples tested indicates that they are valid for the present stock of Corning 512 glass.

It should be noted, however, that some of the values change importantly with slit width, because of lack of symmetry of the absorption bands, and it should be particularly noted that some of the values given are not correct for Corning 592 glasses or for Jena didymium glasses.

12. Infra-Red Dispersion and Absorption of Some Related Compounds. ARCHIE I. MAHAN, *Georgetown University.*

Little work has been done on the dispersion of liquids in the infra-red region, so some additional investigations were undertaken. Pittman earlier studied the dispersion of $CHCl_3$ and $CHBr_3$ and these measurements are concerned with the compounds next in series, namely CH_2Cl_2, CH_2Br_2, and CH_2I_2. The methods used consisted of the utilization of two spectrometers in series, the first serving as the monochromator for the second. The first spectrometer contained a rocksalt prism and the second a hollow prism into which the liquids for study were inserted. Due to the low transmission of the liquids in the vicinity of the absorption band, amplification methods were used to record the presence of energy on the thermocouple surface. The liquids were inserted in the hollow prism and the angles of deviation measured for each wave-length. Knowing the angle of the prism, it was then possible to evaluate the refractive index as a function of the wave-length. It was possible within the resolving power of the apparatus to follow the dispersion through the region of the C−H absorption band near three microns for each of the compounds. The absorption was then followed for each of the compounds, and after making corrections for losses of reflection at the surfaces, the strengths of the bands were compared for all the compounds in the region of the three micron band previously mentioned.

A. H. Pfund, J. Opt. Soc. Am. **25**, 351, 1935.
M. A. Pittman, J. Opt. Soc. Am. **29**, 358 (1939).
K. F. Herzfeld, J. Opt. Soc. Am. **29**, 355 (1939).

13. Whiteness of Light Surface-Colors. DEANE B. JUDD, *National Bureau of Standards.*

The color of a diffusing and highly reflecting surface may depart from white in two characteristic ways; it may fail to be white by being grayish, or it may fail by being chromatic. If these departures are not pronounced, there is frequently a commercial interest in determining the whiteness of the surface, that is, the degree of approach to ideal white taken as the color of a perfectly reflecting, perfectly diffusing surface. Magnesium oxide[1] is taken as the working standard. Whiteness, W, is defined with reference to the color departure of the sample from the standard white, ΔE_s, and the departure of ideal black from white, ΔE_{bk}, by the relation: $W \equiv 1-\Delta E_s/\Delta E_{bk}$. By means of a formula for color differences already validated[2] for surface colors of more than medium lightness, this definition leads to the relation:

$$W=1-0.10(10)^{\frac{1}{2}}[A^{\frac{1}{2}}\langle\Delta S\rangle^2\times 10^4+10(1-A^{\frac{1}{2}})^2]^{\frac{1}{2}},$$

where A is apparent luminous reflectance of the sample for 45-degree illumination and perpendicular viewing, and $\langle\Delta S\rangle^2$ is the sum of the squares of the trilinear coordinate

(r, g, b) differences between the working standard white and the sample on the uniform-chromaticity-scale system.[3] The constant expressing the relative importance of the lightness and chromaticity components of the color departure of the sample from the standard white has been evaluated from the data taken by MacAdam[4] on laundered white goods and from further whiteness estimates by 15 experienced graders of near-white papers.[5] Over the color range studied this formula agrees fairly well with an empirical formula proposed by MacAdam to represent his results, and, because of its more fundamental derivation, the present formula is thought to yield more reliable estimates of whiteness for light colors outside the range studied.

[1] "Preparation and colorimetric properties of a magnesium-oxide reflectance standard," Nat. Bur. Stand. Letter Circular, LC-547 (March 17, 1939).
[2] D. B. Judd, "Uniform tolerances for surface-color specification," J. Opt. Soc. Am. **28**, 52 (1938); also "Specification of uniform color tolerances for textiles," Textile Research **9**, 253, 292 (1939).
[3] D. B. Judd, "A Maxwell triangle yielding uniform chromaticity scales," J. Opt. Soc. Am. **25**, 24 (1935); also J. Research Nat. Bur. Stand. **14**, 41 (1935).
[4] D. L. MacAdam, "The specification of whiteness," J. Opt. Soc. Am. **24**, 188 (1934).
[5] D. B. Judd, "A method for determining whiteness of paper," Paper Trade J., Tech. Sec. **100**, 266 (May 23, 1935); **103**, 154 (August 20, 1936).

14. The Optical Properties of Turbid Media. S. Q. DUNTLEY, *Massachusetts Institute of Technology.*

The optical properties of turbid media have been studied by Ryde,[1] by Kubelka and Munk,[2] and by others. The equations developed by Ryde are more general than those of Kubelka and Munk, but they are based on the assumption that the absorption coefficient of the vehicle is the same as the average absorption coefficient for the entire medium. The present paper is concerned with the form taken by these equations when this assumption is avoided. Experimental evidence of the validity of a new equation will be presented.

[1] Proc. Roy. Soc. **A131**, 451 (1931).
[2] Zeits. f. tech. Physik **12**, 593 (1931).

15. A Direct-Reading Photoelectric Colorimeter. B. T. BARNES, *General Electric Company, Lamp Department, Nela Park, Cleveland, Ohio.*

The cell-filter assembly for this instrument is of the four-filter type described earlier.[1] The direct-reading feature is obtained by use of a current balance. For the total current due to all three circuits, tap-off connections to the main circuit of the current balance are made at the ends of a 100-ohm resistor. The resistance in the tap-off circuit may be set at either 0.1 or 0.4 megohm. Balance is secured by adjusting the current in the main circuit. Then this current is left constant and the resistance between the tap-off points is varied until balance is obtained with each colorimeter circuit. The three resistances required for balance, each divided by 100, are the "instrumental" values of the color coordinates of the source. With a colorimeter which was made and adjusted with special care, the errors in color coordinates obtained for fluorescent lamps of every color except green averaged about half as large as those reported previously.[1] With a steady source, the average difference between "repeat" readings of color coordinates was 0.0001. Systematic errors are likely to be several times this value. Since the "effective resistance" across the cells becomes zero when balance is reached, cells in parallel do not shunt one another. Furthermore, the range over which the current from each cell is proportional to the irradiation is about twice as great with zero external resistance as it is with the usual microammeter resistance of several hundred ohms. Thus this colorimeter varies less in accuracy with changes in intensity than our earlier instrument.

[1] J. Opt. Soc. Am. **29**, 448 (1939).

16. Permissible Short Cuts in the Photoelectric Tristimulus Measurement of Color Difference. RICHARD S. HUNTER, *National Bureau of Standards.*

The greatest asset of photoelectric tristimulus colorimetry is the rapidity with which measurements may be made and converted by computation to trilinear coordinates. The greatest drawback of the method is the serious errors which may result in these coordinates from the failure of the source-filter-photo-cell combinations to be spectrally equivalent to the ICI standard observer for colorimetry. However, these errors are small when the specimens measured are compared with a standard spectrally similar to them. Thus, photoelectric tristimulus colorimetry may be used without serious error for the measurement of color differences between spectrally similar specimens. Moreover, several of the operations which are necessary to obtain photometrically accurate values may be omitted in the measurement of color differences without importantly affecting the result. When the multipurpose reflectometer[1] is used for such measurements, one may sometimes safely neglect to correct for scale errors. Since color differences are determined on this instrument chiefly by relative rather than absolute photometric values, one may frequently omit the use of a previously calibrated standard. The accuracy of photoelectric tristimulus measurements of color difference and the precision of these measurements when the short cuts are employed will be illustrated.

[1] R. S. Hunter, "A multipurpose photoelectric reflectometer," J. Opt. Soc. Am. **30**, 536 (1940); also J. Research Nat. Bur. Stand. **25**, 581 (1940), RP 1345.

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Necrology

Calvin Brewster Coulter

CALVIN BREWSTER COULTER, who died May 10, 1940, spent his scientific life in the study of medical problems in terms of their fundamental backgrounds of chemistry and physics. He was born in Austinburg, Ohio on January 3, 1888. After graduation from Williams in 1907, he received an M.A. from Princeton in 1908, and an M.D. from Columbia in 1913. On finishing his medical training he served first as bacteriologist and then as pathologist at the Presbyterian Hospital, New York. Then, during 1917 and 1918 he was overseas with the American Expeditionary Force with a rank of captain, working as Chief of the pathological laboratory of Base Hospital No. 15. After the war Coulter was Van Cott Research Fellow until 1924 at the Long Island College of Medicine. Then for ten years he was Associate Professor of Bacteriology at Columbia University. After a short interval with the Delmar Institute of Public Health at Columbia, he returned to the Long Island College of Medicine where until his death he was Associate Professor of Pathology and Bacteriologist-in-Chief at the Kings County Hospital.

Although Coulter was a physician and a bacteriologist, he was essentially a scientist exploring the newer aspects of his field. In doing so he drew constantly on the methods developed in physical investigation. Among these, spectroscopy delighted him most, and he used it to study among other things the porphyrins and cytochromes of diphtheria bacilli and their toxins. Coulter possessed great mechanical skill and this served him well in the construction of apparatus. I shall never forget the pleasure with which he built the quartz spectrograph for his work on porphyrins. He even ground and polished the lenses himself.

Aside from his work on the porphyrins, he made critical investigations of the hemolysins and was especially concerned with the spectroscopic properties of the proteins associated with them. In some respects this work was an outgrowth of his early interests in the nature of the electric charge on the red blood cells which he had studied in its bearing on membrane equilibria. For a time he was concerned with bacterial respiration, and especially with the oxidation-reduction potentials of the typhus bacillus. As always, he viewed the problem in terms of its broad biological implications.

—SELIG HECHT

Frederick Eugene Fowle

WITH the passing of Frederick E. Fowle on April 23, 1940, the Federal Government lost a faithful servant and Science a capable researcher on the transmissive properties of the atmosphere. In addition he was a careful compiler and editor of data assembled in the famous *Smithsonian Physical Tables*—a reference work that he kept up-to-date by frequent editing, to include the most recent physical constants of matter.

Born at Arlington, Massachusetts, on November 20, 1869, the son of Frederick E. and Mary Proctor Fowle, the subject of this sketch, F. E. Fowle, as he usually signed his name, was graduated from the Massachusetts Institute of Technology, receiving the B.S. degree in 1894. In the meantime (1887–1894) he was an assistant at the Harvard College Observatory. With this background of training, in 1894, he accepted a position in the Astrophysical Observatory of the Smithsonian Institution,

where he spent some forty-five years in studying various problems connected with the radiation from the sun. He devised methods of measuring the amount of precipitable water in the atmosphere from vapor pressure measurements. His researches on the great transparency of the terrestrial atmosphere, in the region of 8 to 12μ, have been of outstanding value to investigators of incoming planetary radiation, from the Moon, Mars, etc. In addition to the above-mentioned *Physical Tables*, he was co-author in the first five volumes of the *Annals of the Astrophysical Observatory*.

Fowle was proficient in languages and music. He enjoyed playing chess and used to conduct games by correspondence with a friend in South America. Like most scientists he was a member of the usual quota of scientific societies; American Association for the Advancement of Science; American Astronomical Society; American Physical Society; Optical Society of America; Philosophical Society of Washington; and the Washington Academy of Science.

Employed in different, widely separated Federal Departments, each one in his little niche; each one performing his daily task, year-in year-out, life became sort of a routine; and in recent years we saw but little of each other. Hence, early in March, 1940, when I wanted to confer with him regarding atmospheric transparency in the infra-red, it was a distinct shock to learn that he was too ill to be seen. The end came a month later.

He is survived by a wife, Juliana Thomasson, and by two children of a former marriage.

—W. W. COBLENTZ

Dayton C. Miller

DR. DAYTON C. MILLER, internationally renowned physicist of the Case School of Applied Science, Cleveland, passed quietly away at his home on February 22 at the age of 74 years, having suffered a heart attack the day before. At the time of his death he was actively engaged in his laboratory and was about to go to Washington, D. C. to consult with the Ordnance department.

Author of *Sound Waves—Their Shape and Speed; Sparks, Lightning and Cosmic Rays; The Flute and Other Musical Instruments; The Science of Musical Sounds; Anecdotal History of the Science of Sound; The Flute and Flute Playing; Laboratory Physics;* and the *Ether Drift Experiment and Determination of the Absolute Motion of the Earth*, and inventor of the Phonodeik, which turns sound waves into a moving beam of light upon a screen, Dr. Miller was at the same time a modest and friendly teacher, having been connected with Case School for over 50 years and head of the physics department from 1893 to 1936, at which time he was made "Honorary Professor of Physics" for life.

He was a member of the Optical Society of America for many years and was affiliated also with 37 societies in the fields of physics, astronomy, acoustics, mathematics, music, scholarship, and engineering education.

Besides his many degrees, Dr. Miller possessed medals from the Franklin Institute in 1917 and in 1927. In 1925 he won the $1000 prize of the American Association for the Advancement of Science, and in 1928 the City of Cleveland gave him a medal for distinguished service. Mrs. Miller, who survives him, shared with him the distinction of having made an x-ray photograph of her husband, which was the first full skeleton photograph in the world.

—HERBERT ROBINSON YOUNG

Clement Moran

ASSOCIATE PROFESSOR CLEMENT MORAN of the University of New Hampshire was stricken with a heart attack while attending a meeting of the New England section of the American Physical Society on January 26, at the Massachusetts Institute of Technology, and died a few minutes later in the infirmary of that institution.

Professor Moran was born on October 30, 1884, in Black Run, Ohio. He was graduated from Defiance College with the degree of Bachelor of Arts. His first teaching post was at Starkey Seminary, Lakemont, New York. He was appointed an instructor at New Hampshire College in 1914, Assistant Professor in 1921, and Associate Professor in 1928. During the summers of 1917–1919 he was a member of the staff in the standardizing laboratory of the General Electric Company. His second degree was taken at Cornell University, with a thesis on "Effect of the size of ions on the structural temperature of water."

Coming to the University of New Hampshire as a young teacher, he built himself into the organization and spirit of the institution. He was at the same time active in town affairs and campus societies, serving in offices such as town auditor, treasurer of the Red Cross and treasurer of the Society of Phi Kappa Phi. His hobby was photography, which he developed into a useful art for the university extension and for demonstration work. Because of his eminent fairness and his willingness to work long hours with his students, he was very popular with them. The development of the electrical measurements laboratory and, with the late Dr. Willard J. Fisher, that of the sophomore laboratory in engineering physics are memorials to his marked ability in laboratory work.

Professor Moran was married in 1918 to Mary E. Duncan of Hamilton, Ohio, who with her two sons, Harold Duncan Moran and Charles Vernon Moran, survives him.

—HORACE L. HOWES

Albert DeForest Palmer

PROFESSOR ALBERT DEFOREST PALMER was born in Tewksbury, Massachusetts on July 26, 1869. He studied at Brown University, receiving the degree of Ph.B. with the class of 1891. After pursuing graduate work in physics at the Johns Hopkins University from 1891 to 1893 he returned to Brown as Instructor in Physics. He completed his graduate work at Brown and obtained the doctor's degree in 1895. Appointed Associate Professor at

Brown in 1896, he held this post for 38 years until his retirement in 1934. From 1926–1934 Professor Palmer served as Chairman of the Department of Physics.

Professor Palmer's research work was largely in the field of precision measurement and he achieved a considerable reputation for the care and efficiency with which he carried out his program. He worked slowly and published rather infrequently. Nevertheless each paper attested the thoroughness with which he endeavored to attain the maximum precision available in his time. During much of his active career he was interested in optics and among his investigations in this field may be mentioned his measurement of the wave-length of the D_3 helium line (1895), his work on the ellipticity of polarized light produced by strained glass (1921) and his discovery of the effect of electrostatic charge on the polarized light reflected from a metal mirror (1924).

Professor Palmer's interest in measurement led him to write the book *The Theory of Measurement* (1912). It became a standard textbook in its field.

Professor Palmer was a rather shy and extremely modest man who was well liked by all who came to know him intimately. The latter part of his active teaching career was made very trying by almost continual ill-health which however was borne with cheerful endurance. His well-earned retirement was cut short by his death in Pasadena, California, on January 13, 1940.

—R. B. LINDSAY

Thomas Russell Wilkins

THOMAS RUSSELL WILKINS, Professor of Physics at the University of Rochester, died suddenly December 10, 1940 of a heart lesion contracted during the last war. He received his A.B. degree from McMaster University in 1912. In 1917 he enlisted in the United States Signal Corps and was assigned to technical problems related to sound ranging and to pilot balloons. He returned to graduate study and received his Ph.D. degree from the University of Chicago in 1921. From 1918 to 1925 he was Professor of Physics at Brandon College. He spent the year 1925–1926 at Cambridge University with Sir Ernest Rutherford investigating certain radioactive transformations, and came to the University of Rochester as Professor of Physics in 1926. He took an active part in the organization of the Institute of Optics and served as acting director from 1929 to 1938.

In 1930 he undertook a study of photographic effects of atomic particles, and made important contributions to the technique of the study of radioactive processes by this means. Using a specially designed multiplate camera to investigate the scatter of protons by aluminum and magnesium, he observed two distinct particle ranges and correctly interpreted this as due to inelastic scatter. This permitted a direct measurement of the excited states in a stable nucleus, the method proving to be one of unusual refinement. His investigations following this important discovery were cut short by his untimely death, but fortunately plates which he exposed but had not yet measured are being reduced by two of his collaborators. These results will be published shortly.

Russell Wilkins suffered from a certain shyness which he struggled to overcome without ever quite succeeding. Because of this it was necessary to know him well to fully appreciate the kindness and generosity of his character. In his death the University of Rochester suffered a great loss, and one which has been felt keenly by his colleagues.

—BRIAN O'BRIEN